CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER

STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

040-35 - MATHEMATICS

THURSDAY, 24 OCTOBER 2024

1315 - 1615 hrs

Materials to be supplied by examination centres

Candidate's examination workbook Graph paper

Examination paper inserts:

Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

MATHEMATICS

Attempt SIX questions only.

All questions carry equal marks.

Marks for each part question are shown in brackets.

1. (a) THREE mooring lines exert horizontal forces on a bollard, positioned at O, as follows:

32 kN at 40°
18 kN at 65°
25 kN at 110°

The angles are those that the forces make with the real axis Ox.

Determine, *using complex numbers*, the magnitude and direction of the resultant force on the bollard. (8)

(b) Given $Z = \sqrt{2}\angle 45°$, determine $Z + Z^{-1}$ as a complex number in polar form. (8)

2. (a) The crippling load P for a solid steel rod varies directly as the fourth power of its diameter d and indirectly as the square of its length, L.

A steel rod, 2.5m long and 4.5cm in diameter, used as a strut, fixed at both ends, has a crippling load of 235.2 kN.

Determine the crippling load of a similar strut, 3m long and 5cm in diameter. (8)

(b) Express the following function of x as a single algebraic fraction in its simplest form:

$$\frac{14x-6}{2x^2-11x+12}+\frac{4x}{2x-3}-\frac{1}{x-4}$$ (8)

3. The sag, d metres, in a cable of length, L metres, when suspended between two points and subject to a tension, T Newtons, may be determined from the formula:

$$d^2 - \frac{T}{\omega}d + \frac{1}{4}L^2 = 0$$

where ω is the weight in Newtons per metre run.

Calculate, correct to two decimal places, the sag in the cable when $T = 2\ 200$ N, $L = 40$ m and the total weight of the cable = 220 N. (a) (8)

(b) Solve for x in the following equation:

$$\frac{3}{x^2 - 9} + \frac{2}{x - 3} = \frac{3}{x^2 - 2x - 3}$$ (8)

4. (a) Solve EACH of the following for x, correct to 3 decimal places:

(i) $\log_e(e^x + 10) = 3;$ (4)

(ii) $7 = 5e^{-0.4x}$. (4)

(b) Transpose the following formula to make V the subject:

$$t = CR \log_e\left(\frac{E}{V}\right)$$ (4)

(c) Express the following in its simplest form:

$$\frac{\left(a^3b^6\right)^{\frac{2}{3}}}{\sqrt{a^2b^4}}$$ (4)

5. (a) On the same set of axes plot the graphs, in intervals of 1, of

$y_1 = 2x^2 + 5x - 6$ and $y_2 = -x^2 - 2x + 8$ in the range $-4 \leq x \leq 2$.

Additional intermediate points may be plotted to aid the drawing of smooth curves.

Suggested scales: *horizontal axis* 2 cm = 1
vertical axis 1 cm = 1 (12)

(b) Using the graphs plotted in Q5(a), solve, for x and y, the system of equations:

$y = 2x^2 + 5x - 6$

$y = -x^2 - 2x + 8$ (4)

6. Fig Q6 shows a double crank mechanism where AB is the frame.

The distance between the centres A and B is 18 cm.

The crank BC is 27 cm, the crank AD is 36 cm and the link CD is 33 cm.

In the position shown angle BAD is 80°.

Calculate the size of angle ABC for this position. (16)

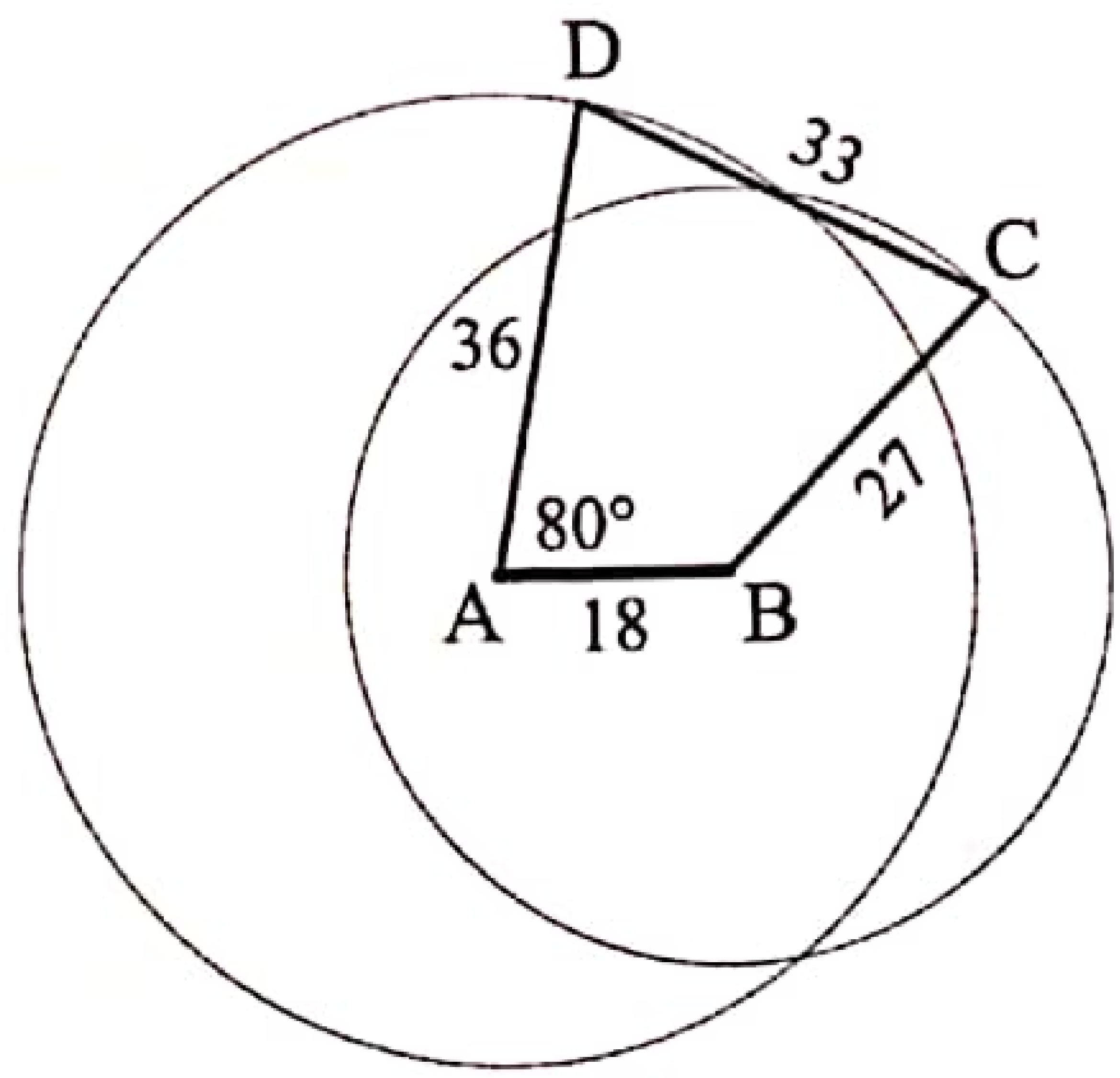

Fig Q6

7. (a) The volume of the square-based glass display case shown in Fig Q7(a) is $864 cm^3$.

The length of the base is x cm and the base is not made of glass.

Determine EACH of the following for the display case:

(i) an expression , in terms of x, for the area of glass, A cm^2 , in the case; (4)

(ii) the dimensions of the case that minimise the amount of glass used; (6)

(iii) the minimum area of glass used. (2)

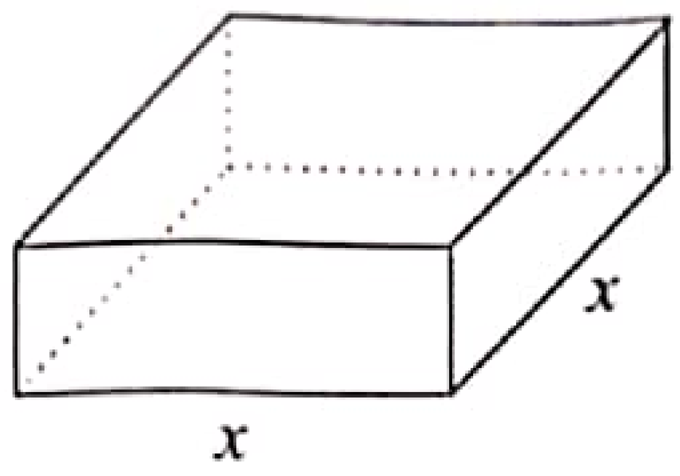

Fig Q7(a)

(b) Determine the first and second derivatives of the following function:

$$u = 3\sin\theta + \frac{1 - 4\sin^2\theta}{1 + 2\sin\theta}$$ (4)

8. (a) The shape of a rugby ball may be represented by the rotation of the shaded area in Fig Q8(a), about the x axis, through one complete revolution.

(i) Determine, using integral calculus, a formula for the volume of a rugby ball in terms of the constants a and b. (10)

(ii) Use the result in Q8(a)(i) to calculate the volume of an adult size rugby ball which has a = 15 cm and b = 9.8 cm. (2)

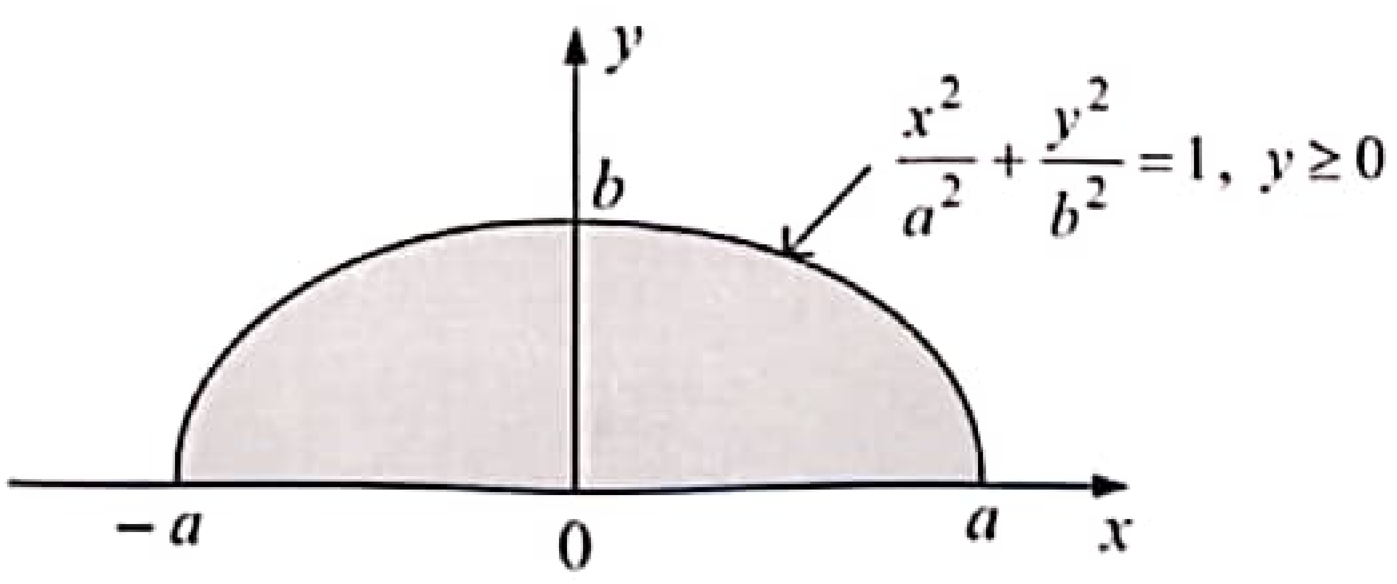

Fig Q8(a)

(b) Evaluate : $\int_0^{\frac{\pi}{6}} \left(\frac{2\sin\theta}{\tan\theta}\right) d\theta.$ (4)

9. (a) The logic circuit shown in Fig Q9(a) has three inputs A, B and C, and one final output X.

Produce EACH of the following for this circuit:

(i) a Boolean expression for output X in its simplest form; (3)

(ii) the truth table, including columns for A, B, C,D,E and X; (3)

(iii) the equivalent logic circuit using only NAND gates (*crossing out any redundant gates*). (4)

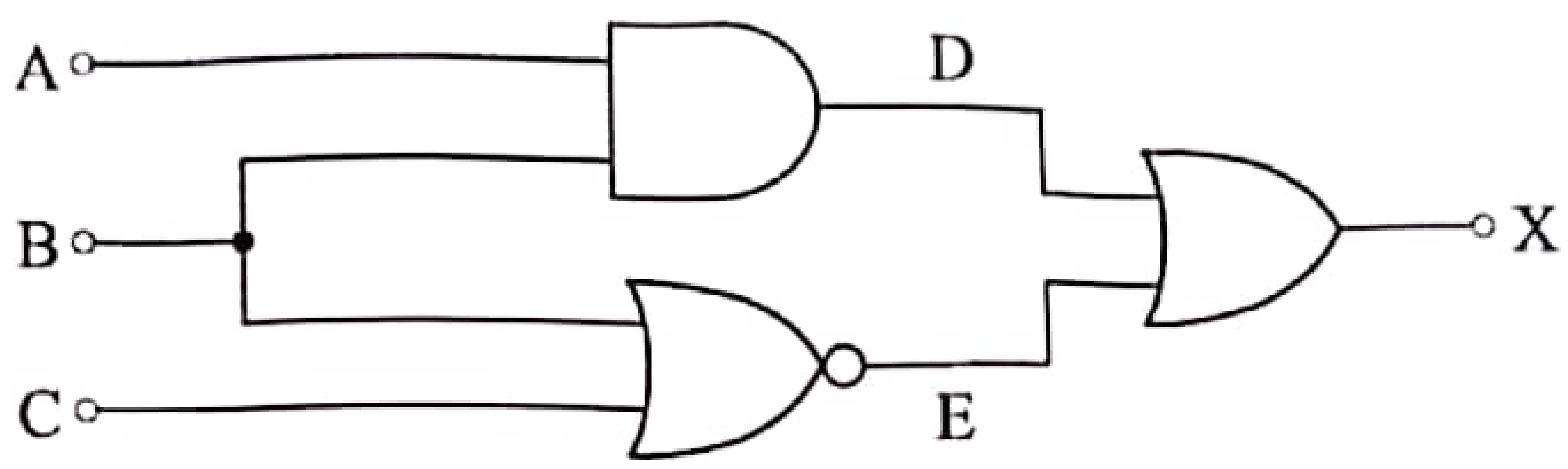

Fig Q9(a)

(b) Determine EACH of the following, *without using a calculator conversion function*:

(i) the binary operation: 10101 × 1011; (2)

(ii) the hexadecimal operation: DA9C – AE4F; (2)

(iii) the conversion of $FADE_{16}$ to binary. (2)