# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
# MARINE ENGINEER OFFICER

## STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

**040-35 - MATHEMATICS**

**THURSDAY, 14 DECEMBER 2023**

**1315 - 1615 hrs**

Materials to be supplied by examination centres

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|---|
| Candidate's examination workbook<br>Graph paper |

Examination paper inserts:

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Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

**MATHEMATICS**

**Attempt SIX questions only.**

**All questions carry equal marks.**

**Marks for each part question are shown in brackets.**

1. (a) Given $Z = \dfrac{10\angle 15° + 4\angle 50°}{5\angle 25°}$, determine Z as a single complex number in polar form. (8)

(b) Solve the following complex equation for $a$ and $b$, where $a$ and $b$ are real numbers:

$$\frac{3a + 2b}{2a + j} = \frac{3 + jb}{1 + ja}$$ (8)

2. (a) The hypotenuse of a right angled triangle is 1 cm longer than twice the length of the shortest side.

The other side is 1 cm shorter than twice the length of the shortest side.

Calculate the lengths of the three sides of this triangle. (8)

(b) Solve for $x$ in the following equation :

$$1 + \frac{2}{x-2} + \frac{3}{x+3} = \frac{10}{x^2 + x - 6}$$ (8)

3. (a) Solve the following system of equations for a, b and c:

$$3a + 2b - 3c = 4$$
$$2a - b + 6c = -1$$
$$a + 3b + 2c = 16$$ (8)

(b) The formula $T = \dfrac{T_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ is associated with the study of relativity,

Make c the subject of the formula. (8)

4. (a) The tension in the tight side of a belt, T newtons, passing round a sheave and in contact with the sheave for an angle of $\theta$ radians is given by:

$T = 42.3e^{0.28\theta}$

Determine the value of $\theta$ when T is 68.5N. (6)

(b) Solve for $x$ in the following equation:

$$16^{0.25x^2} = 32^{x-1.2}$$ (6)

(c) Make t the subject of the following formula :

$$i = Ie^{-\frac{t}{CR}}$$ (4)

5. Table Q5 indicates the deflection, d mm, of a beam under loads, W newtons. The deflection is related to the load by the formula $W = kd^n$ where k and n are constants.

(a) Draw a straight line graph to verify this relationship.

| d (mm) | 7.58 | 10.80 | 14.50 | 18.60 | 23.00 | 27.70 |
|---|---|---|---|---|---|---|
| W (N) | 20 | 25 | 30 | 35 | 40 | 45 |

Table Q5

*Suggested scales: horizontal axis* 2 cm = 0.1

*vertical axis* 5 cm = 0.1 (10)

(b) Use the graph drawn in Q5(a) to determine approximate values of k and n. (6)

6. The bearing of a tower from a ship is due east and the angle of elevation from sea level to the top of the tower is 4°.

After sailing 600 metres in a straight line the bearing of the tower is due north and the angle of elevation to the top of the tower is 7°.

Calculate the height of the top of the tower above sea level. (16)

7. A cylindrical container has to have a volume of $250\pi$ cm$^3$ and is to be made of metal weighing 0.2 g/cm$^2$.

Determine EACH of the following for the container:

(a) the dimensions which minimise its weight; (12)

(b) its minimum weight. (4)

8. (a) The work done during an adiabatic expansion, as the volume increases from $V_1$ to $V_2$, follows the law $PV^n = C$, where C and n are constants.

The work done can be represented by the shaded area in Fig Q8(a).

Calculate the work done, in joules, when an amount of steam expands, following the law $PV^{1.3} = C$, from a volume of 0.25 m$^3$ at a pressure of 820 kN/m$^2$ to a volume of 0.6 m$^3$. (12)

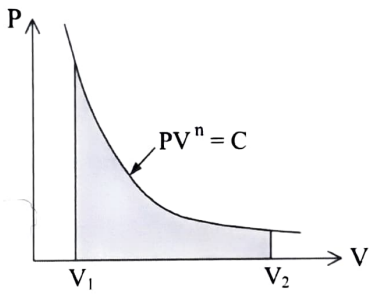


Fig Q8(a).

(b) Evaluate $\int_0^1 \cos\theta \tan\theta \, d\theta$ (4)

9. (a) The truth table for a logic system with inputs A, B and C, and output X, is shown in Table Q9(a).

| A | B | C | X |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

Table Q9(a)

Produce EACH of the following for the logic system:

(i) a Boolean expression in its simplest form; (3)

(ii) the logic circuit with the minimum number of gates; (3)

(iii) the logic circuit using only NAND gates (*crossing out any redundant- gates*). (4)

(b) Determine, *without using a calculator conversion function* the value of $DC_{16} \div 10100_2$, giving the answer in the three forms: binary, hexadecimal and decimal. (6)