CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER

STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

040-35 - MATHEMATICS

THURSDAY, 19 OCTOBER 2023

1315 - 1615 hrs

Materials to be supplied by examination centres

Candidate's examination workbook Graph paper

Examination paper inserts:

Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Solve the following complex equation for a and b, where a and b are real:

   $11a + 3b + j(3a + 2b) = -7 + j(a - 3b + 21)$ (8)

   (b) Given $Z = \dfrac{Z_1 - Z_2}{Z_3 - 2Z_1}$ where $Z_1 = 2 + j5$, $Z_2 = 1 + j3$ and $Z_3 = 9 + j5$

   express Z as a complex number in polar form. (8)

2. (a) The angular velocity, Ω, of a connecting rod may be derived from the formula:

$$\Omega = \frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

   (i) Transpose the formula to make n the subject; (6)

   (ii) Evaluate n when $\Omega = 2.2$ radians per second, $\omega = 10$ radians per second and $\theta = \pi/4$ radians. (4)

   (b) Solve the following system of equations for x and y:

$$x^2 + y^2 = 25$$

$$y = \frac{1}{2}x + 1$$

(6)

[OVER

3. (a) The heat generated by an electric current flowing through a wire varies directly as the time, t seconds, the square of the voltage, V volts, and indirectly as the resistance, R ohms.

When the voltage is 40 volts and the resistance is 70 ohms the heat generated after 15 seconds is 480 units.

Determine the heat generated in 20 seconds when the voltage is 30 volts and the resistance is 42 ohms. (6)

(b) A propulsion problem causes a reduction in a ship's speed of 4 knots throughout a passage of 360 nautical miles, resulting in the ship arriving at its destination 5 hours behind schedule.

Calculate the normal service speed of the ship. (10)

4. (a) Solve the following equation for x:

$\log_e(2 - 3x^2) = -0.6$ (6)

(b) Express the following in its simplest form:

$6\sqrt[4]{16a^4b^8} + 2b\sqrt[3]{27a^6b^3} - 4\sqrt{9a^2b^4}$ (6)

(c) Given $y = \frac{4\log 9}{\log 27 - \log 3}$, determine the value of y without the use of mathematical tables or calculator. (4)

5. The working load limit, *WLL*, for chains of the same grade, and a range of diameters, are given in Table Q5.

(a) Draw a straight line graph to verify that the working load limits, W kg, and the chain diameters, d mm, are related by a law of the form $W = ad^2 + b$ where a and b are constants. (10)

Diameter d (mm)	5	10	15	20	25
WLL W (kg)	200	425	800	1325	2000

Table Q5

Suggested scales: *horizontal axis* 2 cm = 100

vertical axis 2 cm = 200

(b) Use the graph drawn in Q5(a) to determine the value of a and b. (6)

6. (a) The depth of water, h metres, over a sandbar at the mouth of a river on a particular day, is given by:

$$h = 5 + 3\cos\frac{\pi t}{6}$$

where t is the number of hours after local high-water.

Calculate EACH of the following for that day:

(i) the minimum depth of water over the sandbar; (2)

(ii) the time when the minimum depth occurs; (3)

(iii) the latest time, after high-water, when a vessel of draught 4.5 metres may sail over the sandbar with a clearance of 1.3 metres. (5)

(b) TWO radar targets are observed simultaneously at ranges of 5 and 9 nautical miles when the difference in their bearings is 32°.

Calculate the distance between the targets at the time of the observation. (6)

7. (a) Use differential Calculus to determine the coordinates and nature of the stationary points for the function:

$y = 2x^3 - 9x^2 + 12x$. (12)

(b) Given $u = 1 - \frac{2}{t} + \frac{3}{t^2}$ determine $\frac{du}{dt}$ and $\frac{d^2u}{dt^2}$. (4)

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8. (a) A watertight bulkhead can be represented by the area enclosed by the curves:

$y_1 = 8 - 0.01x^2, \quad -10 \le x \le 10$
$y_2 = -0.007x^3, \quad -10 \le x \le 10$
$y_3 = 0.007x^3, \quad -10 \le x \le 10$

as shown by the shaded area in Fig Q8(a).

Calculate the area of the bulkhead, given that the units of length are metres. (10)

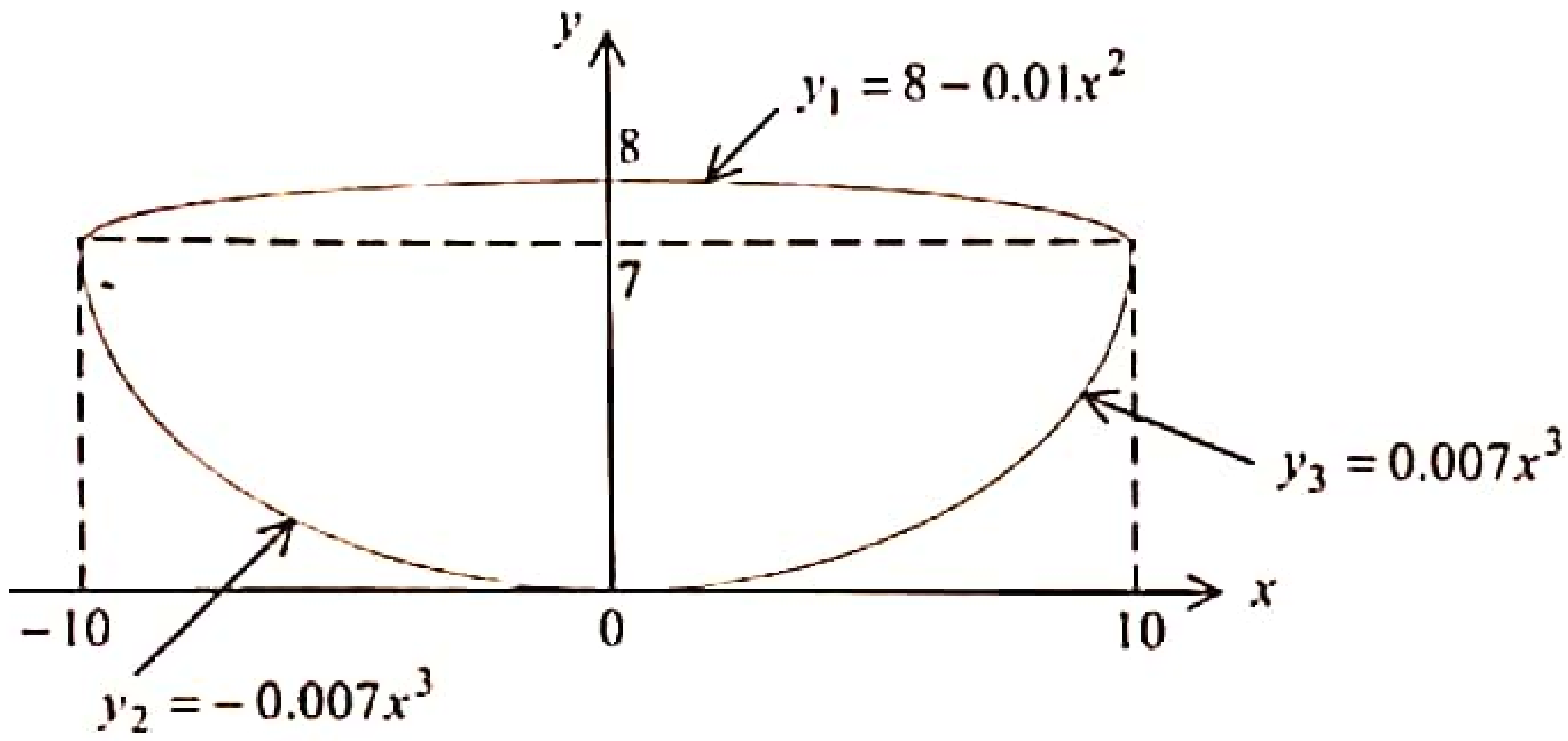

Fig Q8(a)

(b) Evaluate $\int_{0.8}^{1} (2\sin\theta - \cos\theta)\, d\theta.$ (6)

9. (a) The logic circuit in Fig Q9(a) has two inputs A and B, and one output X.

Produce EACH of the following for this circuit:

(i) a Boolean expression for the outputs C, D and X; (3)

(ii) the truth table, including columns for A, B, C, D and X; (3)

(iii) the equivalent logic circuit using only NAND gates (*crossing out any redundant gates*). (4)

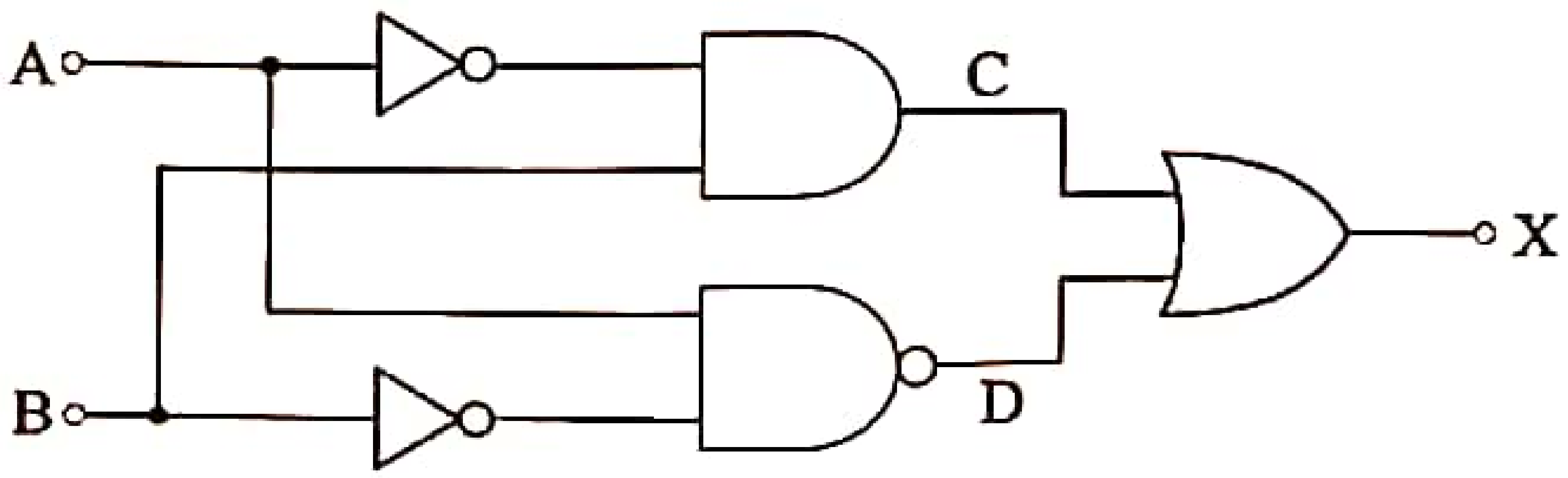

Fig Q9(a)

(b) Determine EACH of the following, *without using a calculator conversion function*:

(i) the binary operation: 10111 × 1101; (2)

(ii) the hexadecimal operation: F7AD + CB9E; (2)

(iii) the conversion : $D3BC_{16}$ to decimal. (2)