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CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER

STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

040-35 - MATHEMATICS

THURSDAY, 20 JULY 2023

1315 - 1615 hrs

Materials to be supplied by examination centres

Candidate's examination workbook Graph paper

Examination paper inserts:

Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Given $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2} + Z_3$, determine Z in Cartesian form when $Z_1 = 1 + j$, $Z_2 = 1 - j2$ and $Z_3 = 0.6 - j1.2$. (8)

(b) Solve the following complex equation for r and θ, where r and θ are real numbers: (8)

$$r\angle\theta^\circ = \frac{6\angle 30^\circ}{1.2\angle 85^\circ} + (3\angle 10^\circ \times 2\angle 25^\circ)$$

2. (a) Solve the following system of equations which model the currents flowing in THREE branches of an electrical network: (8)

$$1.2i_1 - i_2 + i_3 = 2.4$$
$$2.1i_1 - 0.1i_2 - i_3 = 3.1$$
$$0.1i_1 - 2.1i_2 + i_3 = -0.9$$

(b) Solve for x in the following equation: (8)

$$\frac{x}{x+1} - \frac{x+1}{3x-1} = \frac{1}{4}$$

3. (a) The deflection, y, at the centre of a rod under constant load, varies directly as the cube of the length, L, and indirectly as the fourth power of the diameter, d, of the rod.

Calculate the percentage change in y if L decreases by 1% and d increases by 2%. (8)

(b) Make R the subject of the following formula: (6)

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

(c) Fully factorise the following expression: (2)

$$9x^4 - 4x^2$$

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4. (a) A block of metal, initially at 0°C, is placed in a pre-heated oven set at 250°C.

The temperature of the metal, T°C, is given by the function $T = 250(1 - e^{kt})$ where t is the time in minutes the metal has been in the oven and k is a constant.

Given that when $t = 10$, the temperature of the metal is 131.9°C, determine EACH of the following:

(i) the value of k; (6)

(ii) the temperature of the metal when $t = 15$. (2)

(b) Solve the following equation for x, stating the result correct to 3 decimal places:

$$ln\left(1 - x^3\right) = -0.45$$ (4)

(c) Transpose the following equation to make C the subject:

$$t = \frac{1}{M} ln\left(\frac{1}{1+C}\right)$$ (4)

5. (a) Draw the graph of the function $y = \frac{1}{\pi}(2\theta - \sin\theta)$, for the range

$0 \leq \theta \leq 6.4$ radians, in intervals of 0.8 radians.

Suggested scales: *horizontal axis* 2 cm = 1 (12)

vertical axis 5 cm = 1

(b) Using the graph drawn in Q5(a) determine EACH of the following:

(i) the value of θ such that $y = 3.1$; (2)

(ii) the value of y when $\theta = 2$ radians. (2)

6. (a) A jib crane consists of a vertical post AB, 7.2 m in length, the inclined jib BC, 15 m in length and a tie AC. Angle BAC is 125°.

Calculate EACH of the following:

(i) the inclination of the jib to the vertical; (8)

(ii) the length of the tie. (4)

(b) A triangular roofing frame has dimensions as shown in Fig Q6(b).

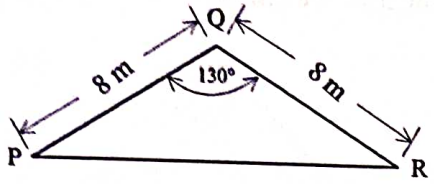

Fig Q6(b)

Calculate the length of PR. (4)

7. (a) An open tank is to be in the shape of a triangular prism, as shown in Fig Q7(a).

The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm.

The tank has a length of l cm and when full of water it is to have a wetted surface area of 10 800 cm^2.

Determine EACH of the following for the tank:

(i) an expression, in terms of x, of its capacity in cm^3; (5)

(ii) the value of x which maximises its capacity; (5)

(iii) the maximum capacity in litres. (2)

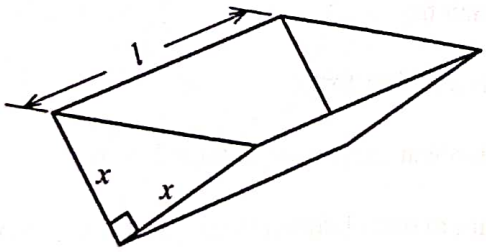

Fig Q7(a)

(b) Determine the first and second derivatives of the function:

$$C = 2r + \frac{5}{r} + 3$$

(4)

8. A church has a grey sandstone wall with THREE arched windows as shown in Fig Q8(b).

EACH of the THREE windows are as shown in Fig Q8(a).

The curved edge of each window is part of the parabola with equation $y = 8x - 2x^2$.

(a) the area of each window; (7)

(b) the area of the sandstone; (5)

(c) the height of the bottom of each window from the ground. (4)

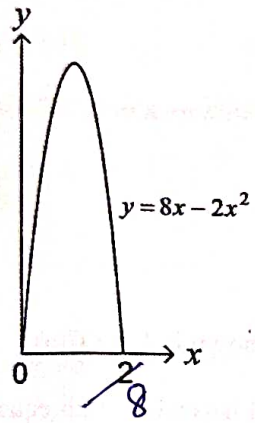

Fig Q8(a)

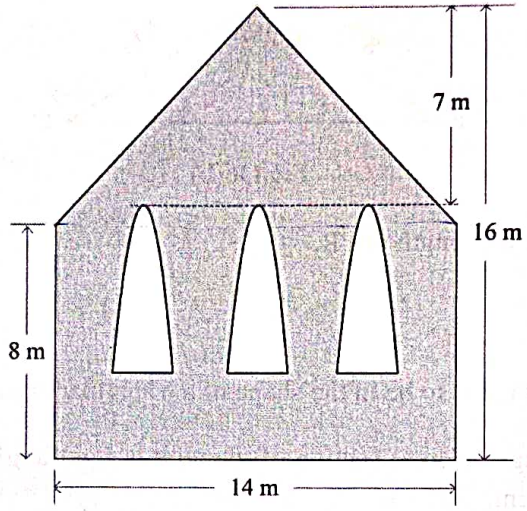

Fig Q8(b)

9. (a) A logic circuit behaves according to the Boolean expression:

$$X = A.B.\overline{C} + \overline{A}.B.\overline{C} + A.\overline{C}$$

Produce EACH of the following:

(i) the truth table for this expression; (2)

(ii) the Boolean expression in its simplest form; (4)

(iii) the circuit diagram for the Boolean expression obtained in (ii); (3)

(iv) the equivalent circuit diagram to that obtained in (iii) using only NAND gates. (3)

(b) Simplify, as fully as possible, the following Boolean expression:

$$\overline{\overline{C + A.B} + \overline{C.\overline{A}}}$$ (4)