MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Forces $F_1 = 5 + j4x$, $F_2 = 3y - j5$, $F_3 = 7x - j17$ and $F_4 = -20 - j5y$ are represented by complex numbers in rectangular form.

Given that the four forces are in equilibrium, determine the value of EACH of the real numbers x and y. (8)

(b) Impedances, $Z_1 = 2 + j5$ and $Z_2 = 4 - j7$ are connected in series to a supply voltage, v, of 230 volts.

Calculate the current, i amperes, as a complex number in polar form, given

$i = \frac{v}{Z}$ where $Z = Z_1 + Z_2$. (8)

2. (a) The square of the time of a planet's revolution in its orbit around the sun varies directly as the cube of its distance from the SUN.

Given that the Earth is 149.6 million km from the SUN and the Earth revolves around the SUN in 365.25 days, determine the time of revolution of Venus, 108.2 million km distant from the SUN, in its orbit around the SUN. (8)

(b) Express the following in its simplest form:

$$\frac{8x^2 - 2y^2}{2x + 6y} \times \frac{x^2 + 6xy + 9y^2}{2x^2 + 5xy - 3y^2}$$ (8)

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3. (a) The impedance, Z, in an a.c. circuit is given by the formula:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Transpose the formula to make C the subject. (6)

(b) Solve EACH of the following for x:

(i) $\frac{x+6}{4} + \frac{2x}{5} + \frac{3-3x}{8} = \frac{3-x}{16}$; (4)

(ii) $12x^2 + 7x - 12 = 0$; (4)

(iii) $x^2 = 10$. (2)

4. (a) Given $V = Ae^{kt}$, where A and k are constants, determine the values of A and k if V = 230 volts when t = 0 and V = 803 volts when t = 10 seconds. (6)

(b) Express the following in its simplest form:

$$\left(\frac{a^{\frac{1}{3}}}{b^2}\right)^{\frac{3}{4}} \times \left(\frac{b^{\frac{3}{4}}}{a^{-\frac{9}{8}}}\right)^{\frac{2}{3}}$$ (6)

(c) Solve the following equation for h:

$$\sqrt{h^5 - 18} = 15$$ (4)

5. (a) Draw the graph of the function $y = 5 + 0.5x - x^2$ for the range $-3 \le x \le 3$ in intervals of 1. (8)

Suggested scales: *horizontal axis* 2 cm = 1

vertical axis 2 cm = 1

(b) Use the graph drawn in Q5(a) to determine EACH of the following:

(i) the maximum value of y; (1)

(ii) the solution of the equation $3.5 + 0.5x - x^2 = 0$; (3)

(iii) the solution of the equation $x - 2x^2 = -13$. (4)

6. A tower stands at the top of a 15° incline.

From a point A on the incline, the angle of elevation to the top of the tower is 40° from the horizontal.

At a point B, which is 100 metres further down the incline in the same vertical plane, the angle of elevation to the top of the tower is 30° from the horizontal.

Calculate the height of the tower. (16)

7. (a) The weekly profit, £P, of a microbrewery is given by:

$P = 120x - 0.4x^2$, where x is the number of barrels of beer produced.

Determine EACH of the following:

(i) the number of barrels of beer to produce per week to maximise the profit; (6)

(ii) the maximum weekly profit. (2)

(b) Given $H = 4t^2 + log_e t - \frac{2}{t}$, determine EACH of the following :

(i) $\frac{dH}{dt}$; (4)

(ii) $\frac{d^2H}{dt^2}$. (4)

8. (a) The angular velocity, ω radians per second, of a flywheel under a constant braking torque, τ newton metres, is given by:

$I\frac{d\omega}{dt} + \tau = 0$, where I is its moment of inertia.

The time in seconds that lapse since the brake is applied is t.

Given that at time $t = 0$, $\omega = \omega_0$, determine EACH of the following for a flywheel:

(i) an expression for ω in terms of t ; (6)

(ii) the time to bring to rest from a speed of 60π rads s^{-1} , when I is 110 kgm^2 and τ is 85 Nm. (4)

(b) Evaluate $\int_{0.2}^{1.0} \frac{400}{V^{1.4}}\, dV$ (6)

9. (a) The truth table for a logic system with inputs A, B and C, and output X, is shown in Table Q9(a).

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Table Q9(a)

Produce EACH of the following for the logic system:

(i) a Boolean expression in its simplest form; (4)

(ii) the logic circuit, with the minimum number of gates; (3)

(iii) the logic circuit, using only NAND gates (*crossing out any redundant gates*). (3)

(b) Determine EACH of the following, *without using a calculator conversion function*:

(i) the binary operation $1001101 + 111101$; (1)

(ii) the conversion of 101111000101_2 to hexadecimal; (1)

(iii) the conversion of $CAB1_{16}$ to decimal; (2)

(iv) the hexadecimal operation $BCE3 - A2DB$. (2)