MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Solve the following complex equation for a and b, where a and b are real numbers:

$$a + jb = \frac{9\angle(\pi/3)}{3\angle(\pi/4)\times 2\angle(-\pi/2)}$$ (8)

(b) The resultant impedance , Z, of TWO circuits of impedances Z_1 and Z_2 , connected in parallel, is given by the formula:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Calculate the resultant impedance, expressing it in polar form, when

$Z_1 = 3 + j5$ and $Z_2 = 2 - j3$. (8)

2. (a) Solve the following system of equations for x and y:

$$\frac{3x+2}{4} - \frac{4y+1}{2} = \frac{19}{4}$$

$$\frac{1-2x}{3} + \frac{1-3y}{5} = \frac{16}{15}$$ (8)

(b) Factorise fully EACH of the following:

(i) $15Z^2 + 14Z - 8$; (2)

(ii) $48m^3 - 27mn^2$; (3)

(iii) $5a^2 - ab - 10a + 2b$. (3)

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3. (a) TWO ferries leave the same port at the same time and arrive at the same destination EIGHT minutes apart.

The faster ferry averaged 2 knots more than the other.

The distance between the TWO ports is 24 nautical miles.

Calculate the average speed of each ferry. (10)

(b) Determine the mass of brass alloy, containing 45% copper, which would be required to combine with 12 kg of copper to form a brass alloy containing 60% copper. (6)

4. Solve for x in EACH of the following equations:

(a) $7^{2(x-3)} = 2401 \times 3^{5-x}$; (6)

(b) $2\ln x = \ln(2x+3) + \ln(x-2), x > 0$; (6)

(c) $e^{2x+1} = 25$. (4)

5. The quantity, Q (m^3/s), of water discharged through an orifice under different pressure heads, H (m), are recorded in Table Q5.

(a) Draw a straight line graph to verify that the relationship between H and Q is a law of the form $Q = aH^n$, where a and n are constants. (10)

H	3.5	3.0	2.5	2.0	1.5	1.0	0.5
Q	11.00	10.24	9.41	8.48	7.42	6.14	4.45

Table Q5

Suggested scales with landscape orientation:
horizontal axis 2 cm = 0.1
vertical axis 2 cm = 0.1

(b) Use the graph plotted in Q5(a) to estimate the value of a and n. (6)

6. (a) An idler gear, 12 cm in diameter, is to be fitted between a 40 cm diameter driving gear and a 60 cm diameter driven gear as shown in Fig Q6(a).

The distance between the centres A and B is 55 cm.

Calculate the size of angles ABC and CAB. (8)

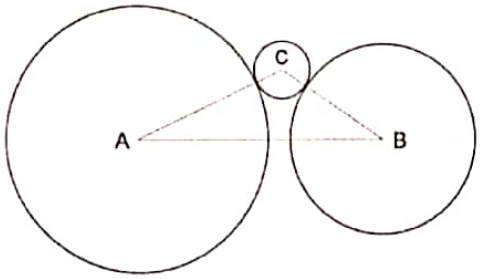

Fig Q6(a)

(b) An engine unit is slung from a horizontal beam by TWO chains 3.0 metres apart.

The lengths of the chains are 2.2 metres and 2.5 metres, and both are hooked to the same lifting eye of the engine unit.

Calculate the angles made by the chains with the beam. (8)

7. A small open rectangular box is to be fabricated from a square sheet of steel of side 20 cm, by cutting congruent squares from each corner and bending the edges up which are then welded along the corner joints.

Given that the volume of the box has to be maximised, calculate EACH of the following:

(a) the side length, using differential calculus, of the square to be cut from each corner; (12)

(b) the volume of the box; (2)

(c) the external surface area of the box. (2)

8. (a) Calculate the shaded area between the curve $y = 8 - x^2$ and the line $y = -2x$, as shown in Fig Q8(a). (8)

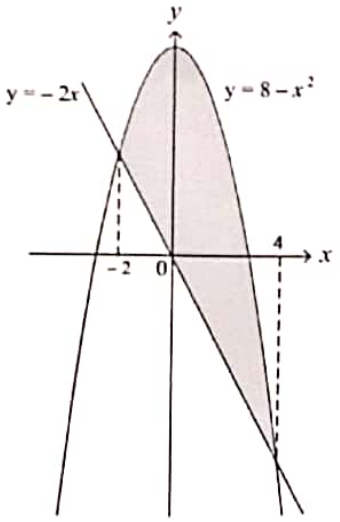

Fig Q8(a)

(b) Given $\frac{ds}{dt} = \frac{2t^3 - 9t^2 + 4t}{2t^2 - t}$, and $s = 3$m when $t = 10$ seconds,

express s as a function of t. (8)

9. (a) An alarm system, X, sends a signal (X = 1) when certain fault conditions are detected in a production process.

The inputs to the system are shown in Table Q9(a).
The alarm system, X returns a value 1 when:

either temperature ≥ 100°C **and** extractor is OFF
or pressure > 8 kPa **and** temperature < 100 °C.

Input	Binary value	Condition
P	1	pressure > 8 kPa
	0	pressure ≤ 8 kPa
T	1	temperature ≥ 100°C
	0	temperature < 100°C
E	1	extractor ON
	0	extractor OFF

Table Q9(a)

Produce EACH of the following for the alarm system:

(i) the truth table; (2)

(ii) the Boolean expression in its simplest form; (3)

(iii) the logic circuit, with the least number of gates possible. (3)

(b) Simplify, as fully as possible, the Boolean expression:

$$\overline{\left(\overline{A.\overline{B}}\right).\left(\overline{B}+C\right)}$$ (3)

(c) Determine, *without using a calculator conversion function*, the value, in hexadecimal form, of $11D_{16} \div 10011_2$. (5)