(6)

# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
# MARINE ENGINEER OFFICER

## STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

**040-35 - MATHEMATICS**

**THURSDAY, 21 JULY 2022**

**1315 - 1615 hrs**

Materials to be supplied by examination centres

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| Candidate's examination workbook<br>Graph paper |

Examination paper inserts:

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Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

**MATHEMATICS**

**Attempt SIX questions only**

**All questions carry equal marks**

**Marks for each part question are shown in brackets**

1. (a) Given $Z = \dfrac{2Z_1 - Z_2}{Z_1 - Z_3}$, where $Z_1 = 4 + j5$, $Z_2 = 3 + j5$ and $Z_3 = 3 + j3$, express Z as a complex number in polar form. (8)

   (b) Two impedances, $Z_1 = 4\angle 30°$ and $Z_2 = 3\angle -10°$ are connected in series to a supply voltage, $v$, of 150 volts.

   Calculate the current, $i$ amperes, as a complex number in Cartesian form, given that $i = \dfrac{v}{Z}$, where $Z = Z_1 + Z_2$. (8)

2. (a) On a particular planet, the height, $h$ metres, of a projectile, $t$ seconds after launching is derived from the formula $h = at + bt^2$ where $a$ and $b$ are constants.

   Given that a projectile on this planet has $h = 296$m when $t = 2$ and $h = 725$m when $t = 5$, determine EACH of the following:

   (i) the values of $a$ and $b$; (7)

   (ii) the duration of the flight. (5)

   (b) Make $S$ the subject of the following formula:

$$T = \sqrt{\frac{2ghDA^2}{d\left(S^2 - A^2\right)}}$$

(4)

3. (a) The lengths, in centimetres, of the sides of a right-angled triangle are $2x - 1$, $3x + 1$ and $3x - 4$.

   Determine the lengths of the THREE sides. (8)

   (b) Given $y = 1 - \dfrac{x^3 - x^2 - 6x}{x^3 + x^2 - 12x}$, express $y$ in its simplest form. (8)

4. (a) The time, $t$ hours, to charge a new tablet to a level C (expressed as a decimal fraction of the battery's full charge) is given by:

$$t = -2.2\ln(1 - C), \quad 0 < C < 1.$$

Determine EACH of the following for the tablet:

(i) the percentage of the full charge achieved after charging for 5½ hours; (6)

(ii) the total time taken, to the nearest ¼ hour, to reach 95% of full charge. (4)

(b) Solve the following for equation $x$:

$$\log_{10}\left(\frac{3}{2x} - \frac{1}{2x^2}\right) = 0$$ (6)

5. (a) Draw the graph of the function $y = 0.5x^2 - 2x - 10$ for the range, $-4 \le x \le 8$, in intervals of 1. (8)

(b) Use the graph drawn in Q5(a) to determine EACH of the following:

(i) the minimum value of $y$; (2)

(ii) the solutions of the equation $0.5x^2 - 2x - 10 = 0$; (3)

(iii) the solutions of the equation $0.5x^2 - 2x - 7 = 0$. (3)

6. (a) A quadrilateral shaped metal plate has dimensions as shown in Fig Q6(a). The angle CDA is 75°.

Calculate EACH of the following for the plate:

(i) the shortest distance from A to C; (4)

(ii) the size of angle DAB. (8)

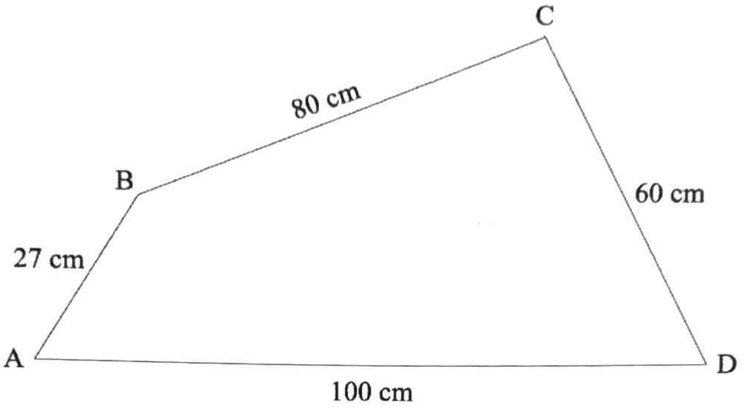


Fig Q6(a) (*Not to scale*)

(b) Solve the following equation for $\theta$ in the range $0° < \theta < 180°$

$\cos 2\theta = -0.75$ (4)

7. (a) A ship sailing at $v$ knots consumes fuel at a rate of $(1.5 + 0.001v^3)$ tonnes per hour.

Determine EACH of the following for this ship when it completes a passage of 2 000 nautical miles at a speed of $v$ knots:

(i) the speed which gives the greatest fuel economy; (10)

(ii) the minimum amount of fuel consumed. (3)

(b) Given $y = \frac{\sqrt{x}}{2} - \frac{2}{\sqrt{x}}$ determine $\frac{dy}{dx}$. (3)

8. A dam is to be built to contain water in a new reservoir.

Relative to axes, as shown in Fig Q8, the sides of the walls can be represented by parts of the graphs of $y = 4x$, $0 \leq x \leq 5$, and $y = 36 - \frac{1}{4}x^2$, $8 \leq x \leq 12$.
The shaded area represents the constant cross-section of the dam wall.

Calculate EACH of the following for the dam wall, given that the dimensions are in metres:

(a) its height; (2)

(b) its cross-sectional area, using integral calculus; (12)

(c) its volume, given that it is 360 m in length. (2)

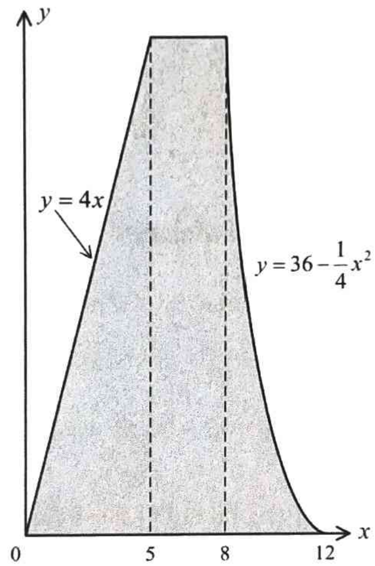


Fig Q8

9. (a) The truth table for a logic system with inputs A, B and C, and output X, is shown in Table Q9(a).

| A | B | C | X |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Table Q9(a)

For this logic system:

(i) produce an unsimplified Boolean expression for output X; (2)

(ii) use a Karnaugh map or Boolean algebra to simplify as fully as possible, the expression for X obtained in Q9(a)(i); (5)

(iii) use the expression for X obtained in Q9(a)(ii) to produce the logic circuit using only NAND gates, *crossing out any redundant gates.* (3)

(b) Determine EACH of the following, *without using a calculator conversion function*:

(i) the binary operation $10111 \times 1101$; (1)

(ii) the conversion of $1101101_2$ to decimal form; (1)

(iii) the conversion of $3AB_{16}$ to binary form; (2)

(iv) the hexadecimal operation AD8F + DECC. (2)