MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Impedances $Z_1 = 5 - j3$ and $Z_2 = 4 + j5$ are connected in parallel to a voltage supply, v, of 240 volts.

   Calculate the current, i amperes, as a complex number in polar form, given that $i = \dfrac{v}{Z}$ where $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$. (10)

   (b) When an a.c. supply of of 200 volts is connected across a particular circuit, the resulting current, i, is $10\angle 60°$ amperes.

   Determine the impedance Z ohms of this circuit, in cartesian form, given that $Z = \dfrac{v}{i}$ where v is the supplied voltage. (6)

2. (a) The amount of energy stored in similar flywheels varies directly as the squares of their speeds and as the fifth power of their diameters.

   A wheel, A, running at its normal operating speed stores 8.2 kJ. Another similar wheel, B, has a diameter 1.7 times that of A and runs at 0.84 of the speed of A.

   Determine the energy stored in B. (6)

   (b) A cable 200 metres long has to be cut into FOUR lengths.

   THREE of the lengths are to be equal and the fourth length must be 10 metres shorter than the sum of the equal lengths.

   Calculate the fourth length. (6)

   (c) The sag, y, at the centre of a wire, of length l, supported at TWO points, distance d apart, is given by the following formula:

   $$y = \sqrt{\frac{3d(l-d)}{8}}$$

   Transpose the formula to make l the subject. (4)

3. (a) Solve the following system of equations for x and y:

$x^2 - 2y^2 + 1 = 0$

$2x + 3y - 1 = 0$ (8)

(b) Solve the following equation for x, $x > 0$, correct to 2 decimal places:

$$\frac{2x+3}{x+1} = \frac{3x-1}{x+2} + 2$$ (8)

4. (a) The strike rate of a ten-pin bowler will increase with practice and this can be approximated to the equation:

$$P = 100\left(1 - 0.5^t\right)$$

where P is the percentage of strikes and t is the number of hours of practice per day.

Calculate EACH of the following:

(i) the number of hours of practice per day, (to the nearest hour), that are likely to be required to achieve a 90% strike rate; (5)

(ii) the likely strike rate, (to the nearest 1%), based on 6 hours practice per day. (3)

(b) Solve the following equation for x, correct to 2 decimal places:

$3^{x^2-1} = 9^{2x}$ (8)

5. (a) A fixed mass of steam was expanded in a turbine.

Values of the pressure P (kPa) and the volume V (m³) of the steam were recorded as shown in Table Q5.

P	10	8	6	4	2
V	0.618	0.744	0.946	1.326	2.363

Table Q5

Draw a straight line graph to verify that the relationship between P and V is the gas law $PV^n = C$, where the expansion index n and C are constants. (10)

Suggested scales: horizontal axis 2 cm = 0.1

vertical axis 2 cm = 0.1

(b) Use the graph drawn in Q5(a) to estimate the values of n and C. (6)

6. TWO lighthouses, L and M, are 12 miles apart on the same stretch of coastline, and M is due south of L.

   At noon a vessel, making good a steady course and speed, bears 150° from L and 105° from M.

   At 1330 hours the vessel is at its closest point of approach to L, which is 5 miles distant.

   Calculate the vessel's course and speed made good. (16)

7. The distance, s metres, moved by a body after t seconds is given by:

   $s = 80t - 8t^2$.

   Determine EACH of the following for the body:

   (a) the initial velocity; (3)

   (b) the velocity after 3 seconds; (1)

   (c) the acceleration after 2 seconds; (2)

   (d) the time when the velocity is zero; (2)

   (e) the distance travelled before coming momentarily to rest; (2)

   (f) the time when passing the starting point; (3)

   (g) sketch $s = 80t - 8t^2$ for $0 \leq t \leq 10$. (3)

8. (a) A gas expands in a cylinder according to the relationship $PV^{1.3} = 398.11$.

   The initial volume of the gas is 0.25 m³ and the work done by the gas during the expansion is 770 J.

   Calculate the final volume of the gas. (8)

   Note: the work done by the gas, W, as it expands from V_1 to V_2 units of volume is:

   $$W = \int_{V_1}^{V_2} P \, dV$$

   (b) Given $\dfrac{dy}{dx} = 9x^2 + 4x - 5 - \dfrac{4}{x^2}$ and $y = 24$ when $x = 2$,

   determine the value of y when $x = 4$. (8)

9. (a) The truth table for a logic system with inputs A, B and C, and output X, is shown in Table Q9(a).

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Table Q9(a)

For this logic system:

(i) produce an unsimplified Boolean expression for output X; (1)

(ii) use a Karnaugh map or Boolean algebra to simplify as fully as possible, the expression for X obtained in Q9(a)(i); (3)

(iii) use the expression for X obtained in Q9(a)(ii) to produce the logic circuit using NOT, AND and OR gates. (3)

(b) Simplify, as fully as possible, the following Boolean expression:

$$\overline{\overline{A}.B}.A$$ (3)

(c) Determine EACH of the following, *without using a calculator conversion function*:

(i) the conversion of 95_{10} to binary; (2)

(ii) the conversion of 11101101100011_2 to hexadecimal; (2)

(iii) the hexadecimal operation $E8BA - BC3D$. (2)