CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER
STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

040-35 - MATHEMATICS

THURSDAY, 25 MARCH 2021

1315 - 1615 hrs

Materials to be supplied by examination centres

Candidate's examination workbook
Graph paper

Examination paper inserts:

Notes for the guidance of candidates:

1. Examinations administered by SQA on behalf of the Maritime & Coastguard Agency
2. Non-programmable calculators may be used.
3. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.
4. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.

Maritime & Coastguard Agency

SQA

MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Marks for each part question are shown in brackets

1. (a) A coil, of resistance R ohms and inductance L henrys, is connected in series with a capacitor of C farads across an a.c. supply of v volts.

Determine EACH of the following for the circuit:

(i) the impedance Z ohms, in complex cartesian form, given that

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where R = 4 ohms, $\omega = 100\pi$, L = 0.04 henrys and $C = 150 \times 10^{-6}$ farads; (4)

(ii) the current, i amps, in polar form, given that $i = \frac{v}{Z}$ and $v = 100$ volts. (6)

(b) The characteristic impedance, Z, of a transmission line is given by

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Determine the condition, in terms of R, L, G and C, that the impedance will be real (6)

2. (a) Solve the following system of equations for x and y: (10)

$$x^2 + 5xy - 4y^2 = -10$$

$$2x + 5y = 1$$

(b) The formula $M = \frac{wx(L - x)}{2}$ gives the bending moment M at a point in a beam.

Calculate the values of x, correct to 3 decimal places, when $M = 65$, $L = 25$ and $w = 2.5$. (6)

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3 (a) A propulsion problem causes a reduction in a ship's speed of 3.5 knots throughout a passage of 630 nautical miles, resulting in the ship arriving at its destination 6 hours behind schedule.

Calculate the passage time. (10)

(b) The minimum diameter, d, of a shaft subjected to a bending moment M and torque T is given by the formula

$$d^3 = \frac{16}{\pi f}\left(M + \sqrt{M^2 + T^2}\right)$$

Transpose the formula to obtain T. (6)

4 (a) Given the formula

$$\frac{T_1}{T_2} = \left[\frac{P_1}{P_2}\right]^{\frac{n-1}{n}}$$

Calculate the value of n when $T_1 = 850$, $T_2 = 340$, $P_1 = 74$ and $P_2 = 1.41$. (8)

(b) Determine the positive values of t which satisfy EACH of the following equations:

(i) $3\log_e\left(t^2 - 1\right) = 6.39$ (4)

(ii) $\frac{1}{1-e^t} = -\cos\frac{\pi}{3}$ (4)

5. (a) The current, i milliamperes, flowing through an electronic component was recorded for a series of applied voltages, v volts, as shown in Table Q5.

Draw a straight line graph to verify that the relationship between i and v is a law of the form $i = av + bv^2$, where a and b are constants. (10)

v	20	40	60	80	100
i	3.6	13.6	30.0	52.8	82.0

Table Q5

Suggested scales with landscape orientation: horizontal axis 1 cm = 5
vertical axis 2 cm = 0.1

(b) Use the graph plotted in Q5(a) to estimate the value of a and b. (6)

8. (a) The shaded area in Fig Q8(a) represents the shape of a prototype aircraft wing.

Calculate the wing area using integral calculus. (8)

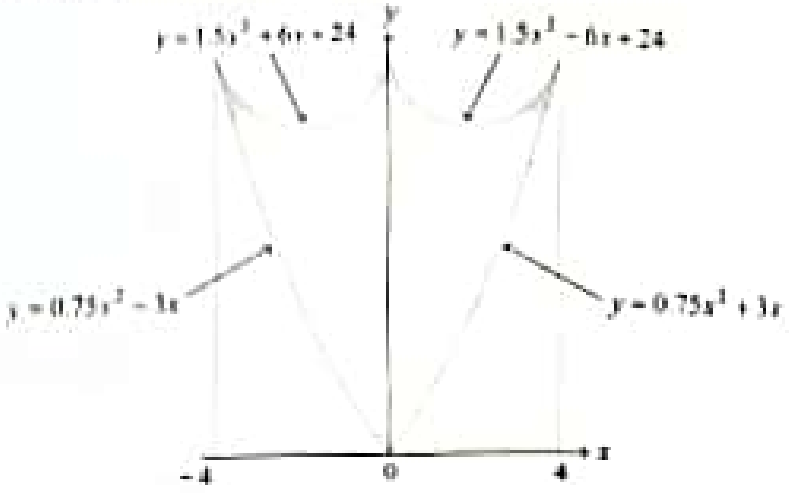

Fig Q8(a)

(b) Evaluate EACH of the following

(i) $\int_{\frac{\pi}{4}}^{2\pi} (4\cos\theta - 5\sin\theta)\,d\theta$ (6)

(ii) $\int_{0}^{2} (\cos^2 x + \sin^3 x)\,dx$ (2)

9. (a) Determine EACH of the following, *without using a calculator conversion function*.

(i) the value, in hexadecimal form, of $FF_{16} + 10001_2$; (5)

(ii) the hexadecimal operation B0DE + CAFE. (2)

(iii) the binary operation 110110 – 101111. (1)

(b) A logic circuit behaves according to the Boolean expression

$$X = \overline{A + B} + \overline{A}(B + C)$$

(i) Without simplification, draw the circuit diagram for the expression using only OR, NOR, NOT and AND gates; (4)

(ii) Simplify the expression as fully as possible. (4)

6 A towboat leaves port A and sails for 3 hours at 15 knots on a course of 120°, it then sails for 6 ¼ hours at 12 knots on a course of 240° to reach port B.

Calculate EACH of the following.

(a) the total distance covered in nautical miles, (3)

(b) the distance of port A from port B, (7)

(c) the bearing of port A from port B. (6)

7 (a) A vessel has a 400 nautical miles passage to make at a constant speed.

At a speed of x knots the cost, in £ per hour, of making the passage is $x^2 + \frac{5400}{x}$.

Determine the speed of the vessel for minimum total cost of the passage. (10)

(b) Given $H = 4\sqrt{t}\left(\frac{8-2t^2}{2+t}\right)$, determine $\frac{dH}{dt}$. (6)

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