

MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

Marks will not be awarded unless relevant working is CLEARLY shown

- 1. (a) Given $Z = \frac{2Z_1 Z_2}{Z_1 + Z_3}$, where $Z_1 = 1 + j3$, $Z_2 = j10$ and $Z_3 = 2 j4$, express Z as a complex number in polar form.
 - (b) Two impedances, $Z_1 = 5 \angle 20^\circ$ and $Z_2 = 4 \angle -15^\circ$ are connected in series to a supply voltage, ν , of 250 volts.

Calculate the current, i amperes, as a complex number in Cartesian form, given that

$$i = \frac{v}{Z}$$
, where $Z = Z_1 + Z_2$.

2. (a) Solve the following equation for x:

$$\frac{x-2}{x+4} - \frac{x+1}{x-4} = \frac{1}{x} \tag{10}$$

(b) Solve the following equation for y:

$$(6y-1)(y+1) = 5 ag{6}$$

3. (a) A white metal for bearings is composed of eleven parts tin, two parts antimony and one part copper by mass.

Determine the masses of antimony and copper required to combine with 220 kg of tin to make white metal.

(i)
$$3ac + 2ad - 6bc - 4bd$$
; (3)

(ii)
$$x^4y - 4x^2y^3$$
;

(iii)
$$2y^3 - 5y^2 + 18y - 45$$
.

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(7)

(8)

(8)

1	(a)	Transpose the following formula to make v the subjection	ect:
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$$S = \frac{1}{k} log_e \left(\frac{L}{L - v^2} \right) \tag{6}$$

(b) Solve the following equation for n:

$$5^{n-2} = 3^{n-1} \tag{6}$$

(c) The time, t seconds, for a voltage to drop from E to V, is given by

$$t = CR \log_e \left(\frac{E}{V}\right)$$

Suggested scales:

Calculate t when
$$C = 3 \times 10^{-6}$$
, $R = 2 \times 10^{6}$, $E = 24$ and $V = 15$.

5 (a) Plot the graph of $y = 3\tan x$, for the range $0 \le x \le 1.2$ in intervals of 0.2. Note that the angle x is in radian measure.

horizontal axis 2 cm = 0.2vertical axis 2 cm = 1

- (b) Using the same axes and scales, plot the graph of $y = \frac{3}{2}\pi 5x$, for the range $0 \le x \le 0.9$, on the graph produced in 0.5(a).
- (c) Use the graphs plotted in Q5(a) and Q5(b) to solve the equation $5x + 3tanx = \frac{3}{2}\pi$ for $0 \le x \le 1.2$

(10)

- (a) A keyway, of width 28 mm, is cut into a stainless steel shaft of diameter 100 mm, as shown in Fig Q6(a).
 - Calculate the shortest distance from the centre of the shaft, A, to the keyway.

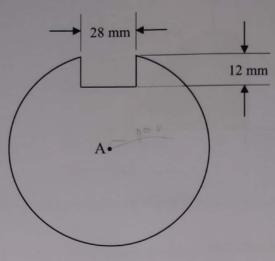


Fig Q6(a)

The depth of water, h metres, over a bar at the entrance to a harbour on a particular day is given by $h = 3 + 2\cos\frac{\pi t}{6}$ where t is the number of hours after local high-water.

Determine EACH of the following for that day:

- (2) (i) the minimum depth of water over the bar;
- (3) (ii) the time when the minimum depth occurs;
- (iii) the latest time after high-water when a fishing vessel of draught 2.6 metres may (3) leave the harbour with a clearance of 1 metre over the bar.
- The annual cost C, in £millions, of pumping water at a hydroelectric power station is 7. given by $C = r + \frac{25}{4r} + 6$, where r is the radius of the supply pipe in metres. Determine the radius of pipe which minimises the annual cost.

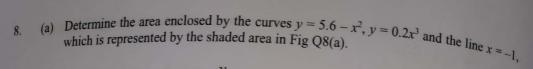
The temperature T °C at a certain location, t hours after midday is given by

$$T = \frac{t^3}{3} - 4t^2 + 15t + 12$$
(8)

Using differential calculus, determine when the temperature starts to fall.

(8)

(8)



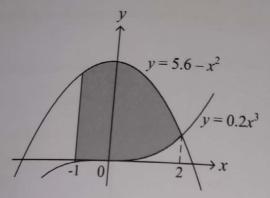


Fig Q8(a)

(b) Evaluate
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (2\cos\theta - \sin\theta) d\theta$$

(6)

9. (a) The truth table for a logic system with inputs A, B and C, and output X, is shown in Table Q9(a).

A	В	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0 0
1	1	1	0

Table Q9(a)

(i) Use Table Q9(a) to produce an unsimplified Boolean expression for X in terms of A, B and C.

(3)

(ii) Use Boolean algebra or a Karnaugh map to show that $X = \overline{A}.B + \overline{A}.C$

(3)

(iii) Produce the logic circuit for the system using NOT, AND and OR gates.

(4)

- (b) Determine EACH of the following, without using a calculator conversion function:
 - (i) the binary operation 10111 x 1101;

(2)

(ii) the hexadecimal operation BEFA + C4D7;

(2)

(iii) the conversion of A3DC₁₆ to decimal.

(2)