

**CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY -  
MARINE ENGINEER OFFICER**

**STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)**

**040-35 - MATHEMATICS**

**THURSDAY, 17 OCTOBER 2019**

**1315 - 1615 hrs**

Materials to be supplied by examination centres:

Candidate's examination workbook  
Graph Paper

Examination Paper Inserts:

Notes for the guidance of candidates:

1. Examinations administered by the SQA on behalf of the Maritime & Coastguard Agency.
2. Candidates should note that 96 marks are allocated to this paper. To pass, candidates must achieve 48 marks.
3. Non-programmable calculators may be used.
4. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

# MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

Marks will not be awarded unless relevant working is CLEARLY shown

- ✓✓ (a) Given  $Z_1 = 3a + j5a$  and  $Z_2 = 11b + j2b$ , solve the following complex equation for  $a$  and  $b$ , where  $a$  and  $b$  are real numbers:

$$Z_1 + Z_2 = -7 + j21 \quad (8)$$

- ✓ (b) Three mooring lines exert horizontal forces on a bollard, positioned at O, as follows:

20 kN at  $30^\circ$

16 kN at  $70^\circ$

10 kN at  $85^\circ$

The angles are those that the forces make with the real axis Ox.

Determine, using complex numbers, the magnitude and direction of the resultant force on the bollard. (8)

2. (a) The force,  $F$ , produced on a ship's rudder is proportional to the area  $A$ , of the rudder, the square of the ship's speed,  $V$ , in knots, and the sine ratio of the rudder angle,  $\theta$ .

For a ship travelling at 12 knots, with a rudder area  $30 \text{ m}^2$  operating at an angle of  $30^\circ$ , the rudder force is 320 kN.

Calculate the force on a similar rudder of area  $25 \text{ m}^2$  operating at an angle of  $22^\circ$  when the ship's speed is 16 knots: (8)

✓ (b) 
$$W = \frac{4 - \frac{8}{x+1}}{x - \frac{2}{x+1}}$$

- (i) Express  $W$  as a single algebraic fraction in its simplest form. (6)

- (ii) Solve for  $x$  when  $W = 3$ . (2)

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- ✓✓ (a) Solve the following system of equations for  $x$  and  $y$ :

$$12x^2 + 16y^2 = 13$$

$$4y = 2x + 3$$

(10)

- ✓ (b) Transpose the following formula to make  $A$  the subject:

$$T = \sqrt{\frac{2ghDA^2}{d(S^2 - A^2)}}$$

(6)

4. ✓ (a) The tension in the tight side of a belt,  $T$  Newtons, passing round a particular sheave and in contact with the sheave for an angle of  $\theta$  radians is given by:

$$T = 63.5e^{0.13\theta}$$

Determine the value of  $\theta$  when  $T$  is 84.2 N.

(6)

- ✓ (b) Solve the following equation for  $x$ :

$$3^{x^2} = 81^{2x-4}$$

(6)

- ✓ (c) Simplify the following as fully as possible:

$$\frac{(2a^3b^2c^4)^2}{(a^9b^6c^3)^{\frac{2}{3}}}$$

(4)

5. (a) On the same set of axes plot the graphs, in intervals of 0.5, of  $y = (x - 1)^2 - 2$  in the range  $-1.5 \leq x \leq 3$  and  $y = 2 - x^2$  in the range  $-2 \leq x \leq 2.5$ .

*Suggested scales:* horizontal axis 2 cm = 1  
vertical axis 2 cm = 1

(12)

- (b) Using the graphs plotted in Q5(a), solve the system of quadratic equations:

$$y = (x - 1)^2 - 2$$

$$y = 2 - x^2$$

(4)

6. A 14 metre tall building has dimensions as shown in the end elevation in Fig Q6. The building has a horizontal base and the walls are vertical.

Calculate the size of EACH of the angles  $x^\circ$ ,  $y^\circ$  and  $z^\circ$ .

(16)

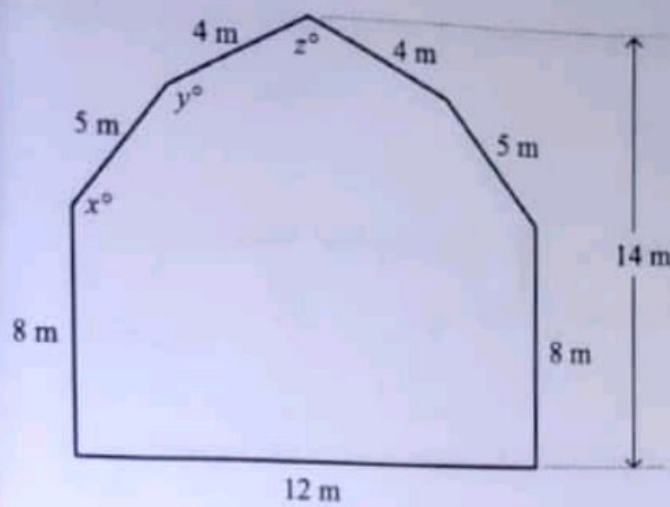


Fig Q6

7. A drip tray is to be fabricated from a thin rectangular steel plate, 36 cm x 24 cm, by cutting squares of side  $x$  cm from each corner and folding up the edges which are then welded up along the corner joints.

The fold lines are shown by the dotted lines in Fig Q7.

- (a) Determine, using differential calculus, the value of  $x$  which maximises the capacity of the tray. (14)
- (b) Calculate the maximum capacity of the tray. (2)

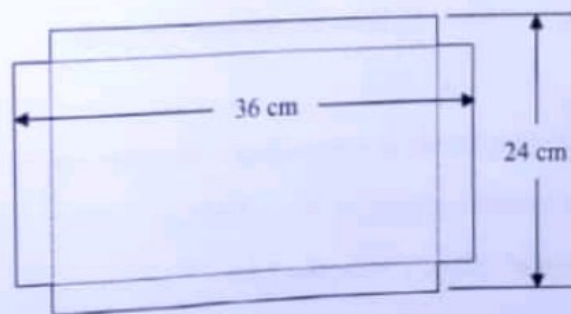


Fig Q7

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8. (a) A gas expands in a cylinder according to the relationship  $PV^{1.3} = 1362$ .  
 The initial volume of the gas is  $0.06 \text{ m}^3$  and the final volume of the gas is  $0.095 \text{ m}^3$ .  
 Calculate the work done by the gas during the expansion. (8)

Note: the work done by the gas as it expands from  $V_1$  to  $V_2$  units of volume is  $W$

$$\text{where } W = \int_{V_1}^{V_2} P dV$$

- (b) Given  $\frac{dy}{dx} = 6x^2 + 2x - \frac{1}{3} + \frac{3}{x^2}$  and  $y = 60$  when  $x = 3$ ,  
 determine the value of  $y$  when  $x = 1$ . (8)

9. (a) The logic circuit in Fig Q9(a) has inputs A and B, and output X:  
 (i) Produce the truth table for this circuit; (3)  
 (ii) State a Boolean expression for X; (1)  
 (iii) State the type of logic gate which produces the same output as this circuit. (1)

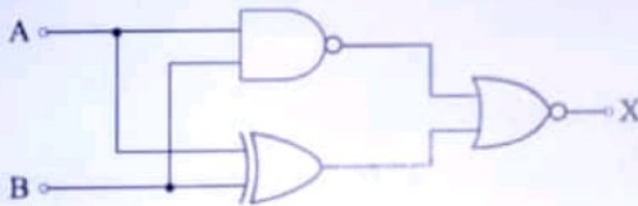


Fig Q9(a)

- (b) Simplify, as fully as possible, the following Boolean expression: (4)

$$\overline{(A + \overline{B})} \cdot \overline{A}$$

- (c) Determine EACH of the following, without using a calculator conversion function:  
 (i) the value, in hexadecimal form, of  $FC_{16} \div 10101_2$ ; (5)  
 (ii) the conversion of  $BDAC_{16}$  to decimal form. (2)