Mathematics December 2018

MATHEMATICS

Attempt SIX questions only

All questions carry equal marks

Marks for each part question are shown in brackets

1. Given $Z_1 = 1 + j2$, $Z_2 = 2 - j$, $Z_3 = 1.5 - j4.5$ and $Z = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3$ determine EACH of the following:

   (a) Z in Cartesian form; (8)

   (b) Q, in polar form, given $Q = \frac{E^2}{Z}$ when $E = 15\angle 120°$. (8)

. (a) Solve the following system of equations for x and y:

$$\frac{5}{x} - \frac{3}{y} = \frac{13}{8}$$

$$\frac{3}{x} - \frac{5}{y} = \frac{49}{24}$$ (10)

(b) Solve the following equation for x:

$$(2x - 3)(4x + 5) = -9$$ (6)

(a) Express the following function of x as a single algebraic fraction in its simplest form: (7)

$$\frac{2}{x} + \frac{1}{x+1} - \frac{3}{x-1} + \frac{2}{x(x+1)(x-1)}$$

(b) Factorise fully EACH of the following:

(i) $a^3 - ab^4$ (3)

(ii) $12x^3 + x^2 - 6x$ (3

(iii) $12p^2 + 36pq + 27q^2$ (

4. (a) Solve for x in the following equation:

$$32^{3x} = 8^{2x+6}$$

(6)

(b) Express the following in its simplest form:

$$\left(\frac{a^{\frac{5}{2}}}{b^{-\frac{5}{3}}}\right)^{\frac{1}{5}} \times \left(\frac{b^{2}}{a^{\frac{3}{2}}}\right)^{\frac{1}{3}}$$

(6)

(c) Transpose the following formula to make t the subject:

$$i = Ie^{-\frac{t}{CR}}$$

(4)

5. Table Q5 shows corresponding measurements of the diameter (d cm) and the breaking load (m kg) of a particular type of rope.

(a) Draw a straight line graph to verify that the diameter and breaking load are related by a law of the form $m = kd^n$ where k and n are constants. (10)

d	2	3	4	5	6
m	80	179	320	501	720

Table Q5

Suggested scales: *horizontal axis* 2 cm = 0.1
vertical axis 2 cm = 0.1

(b) Use the graph drawn in Q5(a) to estimate the value of k and n. (6)

At 0800 hours a patrol boat is set on a course to intercept a vessel as soon as possible.

The vessel is 30 nautical miles due east of the patrol boat and is sailing on a steady course of 030° at a speed of 15 knots.

The best speed of the patrol boat is 25 knots.

Calculate the earliest possible interception time, to the nearest minute. (16)

(a) A company produces particular components in batch sizes which are in multiples of a hundred components.

The production cost, C, of x hundred components is given by:

$$C = 2x^3 - 12x^2 - 72x + 500$$

Determine, using differential calculus, the batch size which minimises the cost of production. (8)

(b) Determine the first and second derivatives of EACH of the following functions:

(i) $y = \frac{x^5}{5} + \frac{3}{x}$ (4)

(ii) $S = \cos t + \ln t$ (4)

(a) Determine the volume of solid of revolution when the shaded area shown in Fig Q8(a) is rotated through one complete revolution about the x axis. (10)

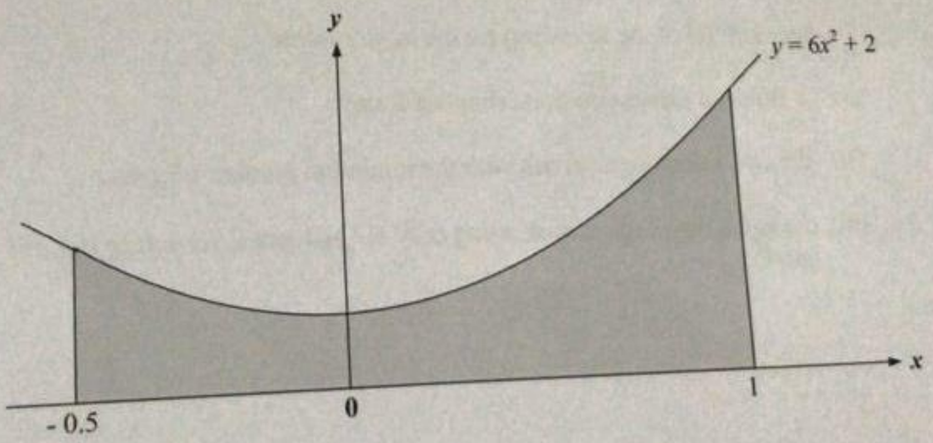

Fig Q8(a)

(b) Evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (\cos\theta - 4\sin\theta)\, d\theta$

9. (a) Determine EACH of the following, *without using a calculator conversion function*:

(i) the conversion of 1275_{10} to hexadecimal; (2)

(ii) the hexadecimal operation EB4C – A5DF. (2)

(b) The truth table for a logic system with inputs P, Q and R, and output X, is shown in Table Q9(b).

P	Q	R	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Table Q9(b)

Produce EACH of the following for the logic system:

(i) a Boolean expression in its simplest form; (4)

(ii) the equivalent logic circuit with the minimum number of gates; (3)

(iii) the equivalent logic circuit using only NAND gates, (*crossing out any redundant gates*). (5)