# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE<br>SCOTTISH QUALIFICATIONS AUTHORITY<br>ON BEHALF OF THE<br>MARITIME AND COASTGUARD AGENCY

STCW 78 as amended MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)
040-35 - MATHEMATICS

THURSDAY, 19 JULY 2018
1315-1615 hrs

Examination paper inserts:
$\square$

Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook

## MATHEMATICS

## Attempt SIX questions only

All questions carry equal marks
Marks for each part question are shown in brackets

1. (a) Given $z=x+j y$, where $x$ and $y$ are real, solve the following equation for $x$ and $y$.

$$
\begin{equation*}
\frac{z}{1-j}+\frac{z}{j}=\frac{20}{3-j} \tag{8}
\end{equation*}
$$

(b) Given $z_{1}=5 \angle 20^{\circ}, z_{2}=4 \angle 30^{\circ}$ and $z_{3}=2 \angle 15^{\circ}$,
express $\frac{z_{1}+z_{2}}{z_{3}}$ as a complex number in polar form.
2. (a) Calculate the mass of a metal containing $55 \%$ nickel which would be required to combine with 10 mg of pure nickel to form an alloy containing $85 \%$ nickel.
(b) Solve EACH of the following equations for $x$ :
(i) $\frac{3}{x+3}+\frac{2}{x-3}=\frac{5}{x-1}$
(ii) $x^{2}-13 x+40=0$
3. Solve for $x$ in EACH of the following equations:
(a) $3^{1-x}=2^{x+1}$
(b) $\ln \left(\frac{3+x}{3-x}\right)=1.25$
4. (a) Solve the following system of equations for $a, b$ and $c$ :

$$
\begin{gather*}
3 a+b-2 c=4 \\
a-2 b+c=6 \\
7 a-6 b-c=10 \tag{10}
\end{gather*}
$$

(b) The sag, $y$ metres, in a cable of length $L$ metres stretched between two supports, $x$ metres apart, as illustrated in Fig Q4(b), is given by the formula:

$$
L=\frac{8 y^{2}}{3 x}+x
$$

Calculate the distance $x$ when $L$ is 75 m and $y$ is 2.4 m .


Fig Q4(b)
5. Variables P and V are thought to be related by a law of the form:
$\mathrm{PV}^{\mathrm{n}}=\mathrm{C}$ where n and C are constants.
Observations of P and V are recorded in Table Q5.
(a) Draw a straight line graph to verify this relationship.

| P | 15 | 20 | 30 | 40 | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| V | 4.42 | 3.55 | 2.60 | 2.06 | 1.74 |

Table Q5

Suggested scales: horizontal axis $2 \mathrm{~cm}=0.1$
vertical axis $2 \mathrm{~cm}=0.1$
(b) Use the graph drawn in Q5(a) to estimate the values of n and C .
6. In the diagram shown in Fig Q6, B and D represent the centres of two gear wheels.

An idler is to be placed at $C$.
Calculate the values of $x$ and $y$.


Fig Q6
7. (a) The rate R , in tonnes per hour, at which a particular vessel consumes fuel is given by: $\mathrm{R}=15+0.00048 V^{3}$, where $V$ is the speed of the vessel in knots.

Determine EACH of the following for this vessel when it embarks on a passage of 500 nautical miles:
(i) the speed of the vessel which minimises the amount of fuel consumed;
(ii) the minimum amount of fuel consumed.
(b) Determine $\frac{d s}{d t}$ given $s=\frac{2\left(t^{2}-t\right)}{\sqrt{t}}$.
8. (a) The work done during an adiabatic expansion follows the law $\mathrm{PV}^{\mathrm{n}}=\mathrm{C}$, where C and n are constants, as the volume increases from $\mathrm{V}_{1}$ to $\mathrm{V}_{2}$.

The work done can be represented by the shaded area in Fig Q8(a).
An amount of steam expands so as to satisfy the law $\mathrm{PV}^{1.13}=\mathrm{C}$.
Calculate the work done, in Joules, when the steam expands from a volume of $0.2 \mathrm{~m}^{3}$ at a pressure of $850 \mathrm{kN} / \mathrm{m}^{2}$ to a volume of $0.5 \mathrm{~m}^{3}$.


Fig Q8(a)
(b) Evaluate $\int_{0}^{\frac{\pi}{2}} \cos x d x$
9. (a) Determine EACH of the following, without using a calculator conversion function:
(i) the binary operation $11011 \times 1011$;
(ii) the hexadecimal operation $\mathrm{BC} 7 \mathrm{E}-9 \mathrm{ADF}$;
(iii) the conversion of $\mathrm{DC}^{2} \mathrm{~B}_{16}$ to decimal;
(iv) the conversion of $1110111111_{2}$ to hexadecimal.
(b) A logic circuit behaves according to the Boolean expression:

$$
\mathrm{X}=\overline{\mathrm{A} \oplus \mathrm{~B}+\overline{\mathrm{A} \cdot \mathrm{~B}}}
$$

(i) without simplification, draw the circuit diagram for the expression, using only XOR, NAND and NOR gates;
(ii) simplify the expression as fully as possible.
(c) Simplify the following Boolean expression as fully as possible:

$$
\begin{equation*}
(\bar{C}+D) \cdot(C+D) \cdot \bar{E} \tag{3}
\end{equation*}
$$

