# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE

SCOTTISH QUALIFICATIONS AUTHORITY
ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY

## STCW 78 as amended SECOND ENGINEER REG. III/2 (UNLIMITED)

## 042-23 - MATHEMATICS

THURSDAY, 15 DECEMBER 2016
1315-1615 hrs

Examination paper inserts:
$\square$

Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook
Graph Paper

## MATHEMATICS

## Attempt SIX questions only

## All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) In manufacturing an engineering component the costs of labour and materials are in the ratio 7:3.

The component is sold at $£ 5568$ producing a profit of $16 \%$.

Determine EACH of the following:
(i) the cost of the materials;
(ii) the percentage increase in material costs when labour costs increase by $5 \%$, the percentage profit is $12 \frac{1}{2} \%$ and the selling price is $£ 5832$.
(b) A cable 200 m long has to be cut into four lengths.

Three of the lengths are to be equal and the fourth length must be 10 m shorter than the sum of the equal lengths.

Calculate the length of the longer piece.
2. (a) Solve the following system of equations for $a, b$, and $c$ :
$4 a+6 b-5 c=-3$
$5 a+2 b+3 c=13$
$15 a+4 b-8 c=-16$
(b) Factorise EACH of the following as fully as possible:
(i) $12 \mathrm{r}^{2}+\mathrm{r}-6$;
(ii) $3 x^{3}-12 x y^{2}$
3. (a) The lengths of the sides of a right-angled triangle are $2 x-3,5 x$ and $5 x-1 \mathrm{~cm}$.

Determine the value of $x$.
(b) Solve the following equation for $x, x>0$, correct to 2 decimal places:

$$
\begin{equation*}
\frac{2 x-1}{x+2}=\frac{3 x-2}{x+1}+1 \tag{8}
\end{equation*}
$$

4. (a) A formula associated with a thermodynamic process is given by:
$\frac{T_{1}}{T_{2}}=\left(\frac{p_{1}}{p_{2}}\right)^{\frac{n-1}{n}}$
Calculate the value of $n$ when $T_{1}=740, T_{2}=296, p_{1}=30$, and $p_{2}=1.2$.
(b) Solve for $s$ in the following logarithmic equation:

$$
\begin{equation*}
\ln \left(\frac{4-s}{3-s}\right)=0.75 \tag{8}
\end{equation*}
$$

5. (a) Draw the graph of the function $y=\tan x$ for the range $1.2 \leq x \leq 1.45$ radians, in intervals of 0.05 radians.

Suggested scales: horizontal axis $2 \mathrm{~cm}=0.05$ vertical axis $2 \mathrm{~cm}=1$
(b) By plotting a suitable straight line on the graph drawn in Q5(a), solve the equation:
$4 x=\tan x$
6. A patrol boat is due north of a vessel at a distance of 40 nautical miles.

The vessel is making good a steady course of $120^{\circ}$ at 15 knots.
The patrol boat intercepts the vessel after 3 hours.
Calculate the course and speed made good by the patrol boat.
7. (a) A stainless steel tank is to be fabricated in the shape of a triangular prism with a regular tetrahedron at each end, as shown in Fig Q7(a).

The length of each edge of the tetrahedron is $x$ metres.
The external surface area, $A$, of the tank is given by:
$A=\frac{3 \sqrt{3}}{2}\left(x^{2}+\frac{16}{x}\right)$
Determine EACH of the following for the tank:
(i) $\frac{d A}{d x}$;
(ii) the value of $x$ which minimises the external surface area;

Verify that the result obtained gives minimum surface area.
(iii) the minimum external surface area.

(b) When a flywheel rotates through an angle of $\theta$ radians in $t$ seconds, its angular velocity is given by $\frac{d \theta}{d t} \mathrm{rads} / \mathrm{s}$, and its angular acceleration is given by $\frac{d^{2} \theta}{d t^{2}} \mathrm{rads} / \mathrm{s}^{2}$.
For a certain flywheel $\theta=27 t-3 t^{2}$.
Determine EACH of the following for this flywheel:
(i) the angular velocity when $t=4$;
(ii) the angular acceleration;
(iii) the time that elapses before the angular velocity is zero.
8. An architectural feature of a grey sandstone building is a rectangular wall, $6 \mathrm{~m} \times 5 \mathrm{~m}$, with an arched window.

The curved edge of the window is part of the parabola with equation $y=3 x-\frac{1}{2} x^{2}$, as shown in Fig Q8.

Determine EACH of the following:
(a) the area of the window;
(b) the area of the sandstone;
(c) the distance of the top of the window from the base.


Fig Q8
9. A tiller head fitting for a yacht is to be made from a solid block of stainless steel, density $8 \mathrm{~g} / \mathrm{cm}^{3}$, as shown in Fig Q9.

A hole of diameter 12 mm is drilled horizontally along the axis of curvature of the left hand side of the block.

A tapered square hole is slotted vertically through the block, having sides of 19 mm at the top and 22 mm at the bottom.

The holes are set apart from each other.
Calculate the mass of the fitting.


Front elevation


Plan

Fig Q9

