# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE SCOTTISH QUALIFICATIONS AUTHORITY

ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY
STCW 78 as amended SECOND ENGINEER REG. III/2 (UNLIMITED)

042-23 - MATHEMATICS

THURSDAY, 20 OCTOBER 2016

1315-1615 hrs

Examination paper inserts:
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Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook
Graph Paper

## MATHEMATICS

## Attempt SIX questions only

## All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) Two ships heading on reciprocal courses pass each other.

One ship has a speed of 6.5 knots greater than the other.

Three hours after passing each other the ships are 111.3 nautical miles apart.
Calculate the speed of EACH ship.
(b) A ship's fuel consumption varies inversely as the calorific value of the fuel and directly as the square of the ship's speed.

The ship burns 28 tonnes of fuel per day of calorific value $42 \mathrm{MJ} / \mathrm{kg}$ when sailing at 14 knots.

Determine the daily consumption when the ship is burning fuel of calorific value $44 \mathrm{MJ} / \mathrm{kg}$ and sailing at 17 knots.
2. (a)
$Y=\frac{5 x-10}{2 x^{2}+3 x-2}+\frac{x}{2 x-1}-\frac{4}{x+2}$

Express $Y$ as a single fraction in its simplest form.
(b) Solve the following system of equations for $a$ and $b$ :

$$
\begin{align*}
\frac{a-1}{4}+\frac{b}{3} & =8 \\
5 a-3 b & =20 \tag{8}
\end{align*}
$$

3. (a) The value, $V$, in thousands of pounds, of a propulsion unit after $t$ years is given by:
$V=275 e^{-0.085 t}$
Calculate EACH of the following for the propulsion unit:
(i) the initial value;
(ii) the number of complete years before its value is less than $£ 100 \mathrm{k}$.
(b) Use laws of indices to simplify EACH of the following:
(i) $\left[\frac{75 a^{\frac{3}{8}}}{25 a^{\frac{1}{4}}}\right]^{4}$
(4)
(ii) $\left[\frac{x^{2}}{y^{4}}\right]^{-\frac{1}{2}} \times\left[\frac{y^{3}}{x^{-6}}\right]^{\frac{1}{3}}$
4. (a) Solve the following equation, for $x>0$, correct to 3 decimal places:

$$
\begin{equation*}
8 x=\frac{50+3 x^{2}}{15+x} \tag{8}
\end{equation*}
$$

(b) Transpose the following formula to make $a$ the subject:

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{a^{2}+b^{2}}{g h}} \tag{8}
\end{equation*}
$$

5. The luminosity, $I$, of an electric lamp with varying voltage, $v$, is shown in Table Q5.

| $v$ | 50 | 70 | 90 | 110 | 130 | 150 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I$ | 4.66 | 18.20 | 52.97 | 109.65 | 208.45 | 364.75 |

Table Q5
(a) Draw a straight line graph to show that $I$ and $V$ are related by a law of the form
$\mathrm{I}=\mathrm{av}{ }^{\mathrm{n}}$, where $a$ and $n$ are constants.
Suggested scales: horizontal axis $2 \mathrm{~cm}=0.1$ vertical axis $2 \mathrm{~cm}=0.2$
(b) Using the graph drawn in Q5(a), determine approximate values for $a$ and $n$.
6. (a) Fig Q6(a) represents a load suspended by a jib crane.

The tie rod AC is 10 metres long, AB is 5.5 metres long and angle CAB is $140^{\circ}$.
Calculate EACH of the following:
(i) the length of the jib BC;
(ii) the angle between the jib and the tie rod.


Fig Q6(a)
(b) Given:
$i=\sqrt{3-2 \sin t}$

Determine the smallest positive value of $t$ when $i=2.2$.
Note: the angle is in radian measure.
7. (a) The yearly profit, $P$ thousand pounds made by a company and the amount, $x$ thousand pounds spent on advertising for the year, are related by:
$P=8 x^{3}-x^{4}$
Calculate EACH of the following:
(i) the amount to spend on advertising to maximise the profit;
(ii) the maximum yearly profit.
(b) Determine the first and second derivatives of the function:

$$
\begin{equation*}
v=\frac{1-\sin ^{2} \theta}{1+\sin \theta} \tag{6}
\end{equation*}
$$

8. (a) Use integral calculus to determine the volume of solid of revolution obtained when the shaded area enclosed by the function $y=\sqrt{9-x^{2}}$ and the $x$-axis, as shown in Fig Q8(a), is rotated about the $x$-axis through one complete revolution.


Fig Q8(a)
(b) Evaluate:

$$
\begin{equation*}
\int_{1}^{4} \frac{2.8}{v^{1.4}} d p \tag{6}
\end{equation*}
$$

9. A right solid steel cone, height 15 cm and base diameter 10 cm , is placed base down in a cylindrical container, 16 cm diameter.

Water is poured into the container until the water is halfway up the cone.
The cone is then removed.
Calculate the resulting drop in water level.

