# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE
SCOTTISH QUALIFICATIONS AUTHORITY
ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY

STCW 78 (as amended) SECOND ENGINEER REG. III/2 (UNLIMITED)

## 042-23 - MATHEMATICS

THURSDAY, 14 JULY 2016

1315-1615 hrs

Examination paper inserts:
$\square$

Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook
Graph Paper

## MATHEMATICS

## Attempt SIX questions only

## All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) A 2400 litre water tank can be filled by two supply pipes A and B working together in 30 minutes.

On its own, pipe A can fill the tank in 32 minutes less than pipe $B$
Calculate the rate of flow of water from each pipe.
(b) Make $R$ the subject of the formula:
$T=2 \pi \sqrt{\frac{L}{g}\left(L+\frac{R^{2}}{r^{2}}\right)}$
2. (a) Determine the value of $z,(z>0)$, which satisfies the equation:

$$
\begin{equation*}
\frac{3 z}{z+1}-\frac{2}{z+2}=1 \tag{8}
\end{equation*}
$$

(b) Solve the following systems of equations for $x$ and $y$ :

$$
\begin{align*}
x^{2}+y^{2}+6 x-6 y-7 & =0 \\
y+1 & =2 x \tag{8}
\end{align*}
$$

3. (a) The modulus of rigidity, $G$, is given by: $G=\frac{R^{4} \theta}{L}$, where $R$ is the radius, $\theta$ the angle of twist and $L$ the length.

Calculate the percentage error in $G$ when $R$ is measured $1.5 \%$ too small, $\theta$ is measured $1 \%$ too small, and $L$ is measured $1.6 \%$ too large.
(b) Given $y=32$, solve the following equation for $x$, correct to 3 decimal places:
$y=\frac{25}{3 x}+x$
4. (a) Solve the following equation for $t, 0<t<2$ :

$$
\begin{equation*}
\ln \left(12-3 t^{2}\right)=-0.78 \tag{6}
\end{equation*}
$$

(b) Express the following in its simplest form:
$5 a \sqrt{9 b^{4}}+4 b\left(\sqrt[3]{8 a^{3} b^{3}}\right)-7\left(\sqrt[4]{a^{4} b^{8}}\right)$
(c) Evaluate the following, without the using mathematical tables or calculator:

$$
\begin{equation*}
\frac{\log 27-\log 9}{2 \log 3} \tag{4}
\end{equation*}
$$

5. The current, $i$, in an electrical component, was recorded at regular intervals of time $t$.

The results are shown in Table Q5.
(a) Draw a straight line graph to show that $i$ and $t$ are related by a law of the form
$i=\mathrm{a} e^{-\mathrm{k} t}$ where a and k are constants.

| $t$ | 0 | 2 | 4 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $i$ | 4.95 | 3.39 | 2.27 | 1.52 | 1.01 |

Table Q5
Suggested scales: horizontal axis $2 \mathrm{~cm}=1$

$$
\text { vertical axis } \quad 2 \mathrm{~cm}=0.2
$$

(b) Using the graph drawn in Q5 (a), determine approximate values for a and k .
6. Fig Q6 shows part of a mechanism.
$A B$ is a link 77 cm long which has a block pivoted to each end.
The blocks can slide in grooves as shown.
The point of intersection of the line of centres is at $C$.
Initially, $\mathrm{BC}=52.3 \mathrm{~cm}$ and $\mathrm{AC}=38.5 \mathrm{~cm}$.
Calculate EACH of the following:
(a) the angle between the line of centres (i.e. angle BCA);
(b) the inclination of AB to AC ;
(c) the distance A moves if block B moves 16 cm towards C from the given position.


Fig Q6
7. (a) Use differential calculus to determine EACH of the following for the function $y=x^{3}-3 x^{2}-9 x+10$
(i) the coordinates of the turning points;
(ii) the nature of the turning points.
(b) The area, $A \mathrm{~cm}^{2}$, of a pool of oil under a leaking sump is given by $A=t+\frac{t^{2}}{16} \quad$ where $t$ is the time in minutes.

Calculate EACH of the following for the pool of oil after 20 minutes:
(i) the area;
(ii) the rate the area is growing.
8. (a) The velocity $v$, in $\mathrm{ms}^{-1}$, of a particle at time $t$, in seconds, is given by

$$
v=\frac{d s}{d t}=30-8 t
$$

Given $s=0$ when $t=0$, determine EACH of the following :
(i) $s$ in terms of $t$;
(ii) the distance travelled in 4 seconds from $t=0$.
(b) Integrate EACH of the following functions, with respect to the given variable:
(i) $6 x^{2}+\frac{2}{\sqrt{x}}-3$
(ii) $2 \theta+3 \cos \theta-4 \sin \theta$.
(c) Evaluate $\int_{1}^{2} \frac{4}{x^{2}} d x$
9. A paper cup has internal dimensions: height 12 cm , top diameter 9 cm , and bottom diameter 6 cm , as shown in Fig Q9.

Water is poured into the cup to a depth of 7 cm .
(a) Calculate the surface area of the water.
(b) A sphere submerged in the water increases the depth to 10 cm .

Calculate the diameter of the sphere.


Fig Q9

