# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE

SCOTTISH QUALIFICATIONS AUTHORITY
ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY

## STCW 95 SECOND ENGINEER REG. III/2 (UNLIMITED)

## 042-23 - MATHEMATICS

THURSDAY, 18 OCTOBER 2012
1315-1615 hrs

Examination paper inserts:
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Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook
Graph Paper

## MATHEMATICS

## Attempt SIX questions only

## All questions carry equal marks

Marks for each part question are shown in brackets

1. (a) The normal hourly rate of pay of a certain worker between 9 am and 5 pm is $£ x$.

For each hour he works in the evening he is paid one and a half times his normal rate.

For each hour he works at weekends he is paid twice his normal rate.
In one particular week he works 53 hours. This consisted of 40 hours at the normal daily rate with 5 hours worked in the evening and 8 hours worked at the weekend.

He was paid a total of $£ 584.20$ for that week’s work.

Calculate his normal daily hourly rate of pay.
(b) A formula used in connection with close coiled helical springs is:
$P=\frac{G F d^{5}}{8 h D^{3}}$
Determine the approximate percentage change in $P$ if $d$ is increased by $2 \%$ and $D$ is decreased by $3 \%$.
2. (a) Solve for $x$ in the following equation:
$\frac{3}{x-1}+\frac{3}{x+1}=4$
(b) The perimeter of a square and a rectangle are equal. The length of the rectangle is 11 cm . The area of the square is $4 \mathrm{~cm}^{2}$ more than the area of the rectangle.

Calculate the length of the side of the square.
3. (a) Van der Waal's equation for the pressure, $p$, of a real gas is:
$p+\frac{a}{v^{2}} \quad v-b=R T$
Transpose the terms in the equation to make $p$ the subject.
(b) The resistance, $R$ ohms, of copper wire at $t^{\circ} \mathrm{C}$ is given by:
$R=R_{0} \quad 1+\alpha t$ where $R_{0}$ is the resistance at $0^{\circ} \mathrm{C}$ and $\alpha$ is the temperature coefficient of resistance.
$R=26.48$ when $t=25$ and $R=27.84$ when $t=33$
Calculate the values of $R_{0}$ and $\alpha$.
4. (a) The current, $i$ amperes, flowing through a capacitor at time $t$ seconds is given by: $i=7.51-e^{-\frac{t}{c R}}$ where the circuit resistance, $R$, is $26.8 \times 10^{3}$ ohms and the capacitance, $C$, is $15.25 \times 10^{-6}$ farads.

Determine the time for the current to reach 6.75 amperes.
(b) Solve for $n$ in EACH of the following equations:
(i) $2^{n+1}=7^{2 n-1}$
(ii) $5 \ln 2 n-1=4$
5. (a) Draw the graph of $T$ in the range $1.2 \leq R \leq 2.2$ in intervals of 0.2 .
$T=\frac{10}{R^{2}} \ln R$
Suggested scales $\quad$ horizontal axis $2 \mathrm{~cm}=0.2$
vertical axis $2 \mathrm{~cm}=0.1$
(b) Using the graph drawn in Q5(a), determine EACH of the following:
(i) the value of $R$ such that $T$ is a maximum;
(ii) the maximum value of $T$.
(c) Given $\frac{d T}{d R}=\frac{10}{R^{3}} 1-2 \ln R$

Using this derivative function, determine the value of $R$, correct to three decimal places such that $T$ is a maximum.
6. (a) Fig Q6(a) shows a triangular prism on a horizontal base.

CD is vertical, $\mathrm{AB}=55 \mathrm{~mm}$, angle $\mathrm{DBC}=20^{\circ}$, angle $\mathrm{DAC}=15^{\circ}$ and angle $\mathrm{ACB}=100^{\circ}$

Calculate the vertical height CD.


Fig Q6(a)
(b) Given $\sin \omega t-0.1=0.4$

Calculate the least positive value of $t$ if $\omega=50 \pi$
7. (a) Given $\mathrm{T}=10+5 \sin x-3 \cos x$

Determine the value of $x$ (for $0^{\circ} \leq x \leq 360^{\circ}$ ) such that T has a maximum value.
(b) The displacement $s$ metres of a body from a fixed point is given by the equation: $s=4 t^{3}-3 t^{2}+8 t$ where $t$ is the time in seconds.

Determine EACH of the following:
(i) the times when the velocity of the body is $8 \mathrm{~m} / \mathrm{s}$;
(ii) the acceleration after 1 second.
8. (a) The shading in Fig Q8(a) shows the area enclosed between the functions $y_{1}=8+12 \sin \theta, \quad y_{2}=5 \sin \theta$ and the ordinates $\theta=\frac{\pi}{2}$ and $\theta=\frac{5 \pi}{6}$

Calculate the shaded area.


Fig Q8(a)
(b) Given $\frac{d^{2} T}{d r^{2}}=r^{2}-\frac{1}{r^{2}}$
$\frac{d T}{d r}=2^{1 / 3}$ when $r=1$ and $T=2$ when $r=1$
Express $T$ as a function of $r$.
9. (a) A horizontal cylindrical vessel is 600 mm long and 120 mm in diameter. Liquid is poured into the vessel until the maximum depth of the liquid is 50 mm .

Determine the volume of liquid in the vessel.
(b) A solid right aluminium cone has a height of 250 mm and a base diameter of 200 mm . It is melted down and recast as a solid sphere.

Calculate the diameter of the sphere allowing for $2 \%$ loss in the melting process.

