# CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY MARINE ENGINEER OFFICER 

EXAMINATIONS ADMINISTERED BY THE SCOTTISH QUALIFICATIONS AUTHORITY<br>ON BEHALF OF THE<br>MARITIME AND COASTGUARD AGENCY<br>STCW 95 SECOND ENGINEER REG. III/2 (UNLIMITED)

042-23 - MATHEMATICS

THURSDAY, 21 OCTOBER 2010
1315-1615 hrs

Examination paper inserts:
$\square$

Notes for the guidance of candidates:

1. Non-programmable calculators may be used.
2. All formulae used must be stated and the method of working and ALL intermediate steps must be made clear in the answer.

Materials to be supplied by examination centres:
Candidate's examination workbook
Graph Paper

## MATHEMATICS

## Attempt SIX questions only

## All questions carry equal marks

## Marks for each part question are shown in brackets

1. (a) A ship is scheduled to complete a journey of 540 nautical miles at an average speed of 12 knots. It covers the first quarter of the journey at an average speed of 13.5 knots. However due to a series of factors its speed in the last three quarters of the journey is reduced and it arrives at its destination 1 hour 40 minutes late.

Calculate the average speed of the ship over the second part of its journey.
(b) The wind force $W$ on a vertical surface varies directly as the area, $A \mathrm{~m}^{2}$, of the surface
and directly as the square of the wind velocity, $v \mathrm{~km} / \mathrm{hr}$. When the wind speed is $36 \mathrm{~km} / \mathrm{hr}$ the force on an area of $2.5 \mathrm{~m}^{2}$ is 320 Newtons.

Calculate the force on a surface of area $4 \mathrm{~m}^{2}$ when the wind speed is $60 \mathrm{~km} / \mathrm{hr}$.
2. (a) Solve for $x$ in the following equation:
$\frac{6 x+1}{2 x-3}=\frac{3 x-1}{x-1}$
(b) Solve the system of equations for $A$ in the range $0 \leq A \leq \frac{\pi}{2}$
$4 \sin A+5 \cos A=6.353$
$13 \sin A-8 \cos A=3.752$
(c) Factorise completely:
$2 x^{3} y^{2}+x^{2} y^{3}-6 y^{4} x$
3. (a) The bending moment, $M$, at a point on a beam is given by:

$$
M=\frac{3 x(20-x)}{2}
$$

where $x$ metres is the distance from the point of support to the end of the beam.
Calculate the value of $x$ when the bending moment is 50 Nm .
(b) Transpose the terms in the following equation to make $C$ the subject:

$$
f=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}}
$$

4. (a) Determine the value of $x$ correct to three decimal places that satisfies the following equation:
$0.027^{x-1}=3.26$
(b) The tension, T Newtons, in the tight side of a belt passing round a pulley wheel and in contact with the pulley for an angle $\theta$ radians is given by the equation:
$\mathrm{T}=43.8 e^{0.32 \theta}$

Determine the value of $\theta$ when T is 75 Newtons.
(c) The life expectancy, $N$ years, of a certain machine costing $£ C$, and its value $£ V$ after $n$ years are related by the formula:
$n=\frac{\ln V-\ln C}{\ln \left(1-\frac{2}{N}\right)}$
Calculate the age of the machine which cost $£ 75000$ with a life expectancy of 6 years that has depreciated to a value of $£ 10000$.
5. Table Q5 shows the values of the resistance, R ohms, and the voltage, V volts, recorded in an experiment.
(a) Draw a straight line graph to show that R and V are related by a law of the form $\mathrm{R}=\frac{\mathrm{a}}{\mathrm{V}}+\mathrm{b}$ where a and b are constants.
(b) Determine approximate values of a and b .

| R ohms | 47.4 | 52.2 | 55.9 | 59.9 | 62.1 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| V volts | 0.111 | 0.102 | 0.093 | 0.088 | 0.085 |

Table Q5
Suggested scales: $\quad$ horizontal axis $2 \mathrm{~cm}=0.5$
vertical axis $2 \mathrm{~cm}=2$
6. (a) A tower 55 metres high stands on the top of a hill which has a $10^{\circ}$ incline. The angle of depression from the top of the tower to a point A on the slope is $72^{\circ}$. B is a point further down the slope from A . The angle of depression from the top of the tower to $B$ is $48^{\circ}$. Points A an B and the top of the tower all lie in the same vertical plane.

Calculate the distance between the points A and B.
(b) Given: $2 \tan \theta=\tan \alpha+\tan \beta$

Solve for $\theta$ in the range $0^{\circ} \leq \theta \leq 360^{\circ}$ when $\alpha=30^{\circ}$ and $\beta=45^{\circ}$.
7. (a) The rate at which a particular ship's engine consumes fuel is given by: rate $=30+0.002 v^{3}$ tonnes per hour (where $v$ is the speed of the ship in $\mathrm{km} / \mathrm{hr}$ ).

Calculate EACH of the following:
(i) the speed at which the minimum amount of fuel is used on a voyage of 1500 km ;
(ii) the minimum amount of fuel for the journey.
(b) Determine the first derivative of the following function:
$R=h+4 \sqrt{h}-\frac{3}{h \sqrt{h}}+\frac{1}{h}$
8. (a) Fig Q8(a) shows the graph of the function $y=-\frac{1}{2}\left[x^{4}+x^{3}-2 x^{2}\right]$

Determine the area enclosed by the function and the $x$ axis.


Fig Q8(a)
(b) Evaluate $\int_{1}^{3}\left(\frac{2 x^{2}+1}{x}\right) d x$
9. (a) (i) Determine the mass of a hemispherical copper container whose external and internal diameters are 28 cm and 26 cm respectively.

Note: Copper weighs $8.9 \times 10^{3} \mathrm{~kg}$ per $\mathrm{m}^{3}$.
(ii) The entire surface of the container is given a protective coating.

Determine the total surface area to be covered.
(b) Fig Q9(b) represents a regular pentagon of side 200 mm .

Calculate the shaded area.


Fig Q9(b)

